



 Raising A Mathematician Foundation®

ROOTS 2024

Square Roots

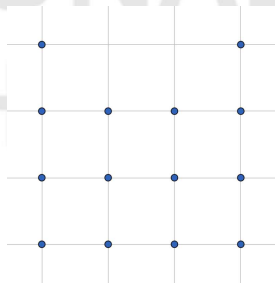
September 22, 2024

Answers

1. Jeff starts at 0 and counts by 3's: 0, 3, 6, 9, 12, ... At the same time and at the same speed, Hurudaya counts backwards from 2024 by fives, 2024, 2019, 2014, ... Find the one number that both Jeff and Hurudaya count at the same time.

Answer: 759

2. How many triangles can be constructed by joining any three of the fourteen dots?



Answer: 332

3. The product of two consecutive integers is divisible by 60. What is the sum of the smallest pair of consecutive integers that satisfies this condition?

Answer: 31

4. Let a , b , c be positive integers such that

$$2^a + 2^b + 2^c = 148$$

Find the value of $a + b + c$?

Answer: 13

5. Given that $a \neq 0$, such that

$$\sqrt{a} + \frac{1}{\sqrt{a}} = 3$$

Find $\left(a - \frac{1}{a}\right)^2$

Answer: 45

6. A rectangle with sides x and y has area 15 sq. units and perimeter 16 units. Find the value of $x^2 + y^2$.

Answer: 226

7. Mayank was born in the 20th century. His age in 2023 is the sum of the 4 digits of his birth year. What is his birth year?

Answer: 1997

8. The first term of a sequence is 2014. Each succeeding term is the sum of the cubes of the digits of the previous term. What is the 2014th term of the sequence?

Answer: 370

9. Find the number of four digit positive integers that contain exactly two distinct digits. (1101 and 9922 are two such integers)

Answer: 567

10. Compute the sum of all $a + b$ where a and b are integers such that $a > b$ and $a^2 - b^2 = 2024$.

Answer: 4032

11. Find the unique integer $CA7DB$ with nonzero digits such that:

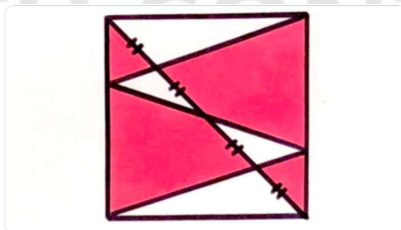
$$ABCD \times 3 = CA7DB$$

Answer: 14739

12. Twenty-five people who always tell the truth or always lie are standing in a queue. The man at the front of the queue says that everyone behind him always lies. Everyone else says that the person immediately in front of them always lies. How many people in the queue always lie?

Answer: 13

13. If the fraction of the square is shaded is $\frac{a}{b}$ in reduced form, find the value of $a + b$.



You may need to use the following results:

- (a) Pythagoras' theorem states that, in a right angled triangle, the square of the length of the hypotenuse (the side opposite to the right angle) is equal to the sum of squares of the lengths of the other two sides.
- (b) If $\triangle ABC$ and $\triangle DEF$ are such that $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$, then $\triangle ABC$ and $\triangle DEF$ are said to be similar and we have that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

Answer: 5

14. A pile of tomatoes is made where the base is in the shape of an equilateral triangle, with the number of tomatoes in the bottom level as 36. In the pocket formed by three tomatoes, lies a tomato in the upper level. If the levels are formed till there is only one tomato in the top layer, how many tomatoes are in the pile?

Answer: 120

15. There is a passenger train that has seats on either side of an aisle (pathway to walk). The number of seats on either side are either equal or one side has an extra seat than the other. Every seat is allotted a number where the first seat is allotted the number 1, its adjacent seat as 2, and so on. Seat #1 is a Window Seat (W). Seats on either side of the aisle are called Aisle Seats (A). Any other is called a Middle seat (M). The seats are numbered in a specific order from left to right or right to left. If you are told that seat number 28 is W, 50 is A, 31 is M, 12 is M, what kind of seat is #24??

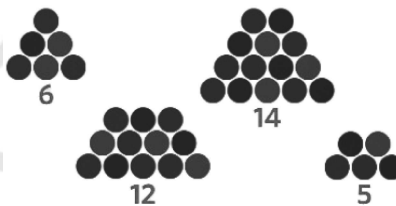
- (a) Window Seat
- (b) Middle Seat
- (c) Aisle Seat
- (d) Insufficient data to solve the problem

Answer: Middle Seat

16. Using the digits 1 or 2, how many numbers can be formed where the sum of the digits is 8?

Answer: 32

17. Let the numbers that can be arranged in the below manner be called as Roof Numbers. How many roof numbers exist between 50 and 300?



Answer: 3

18. Find the least positive number n such that all the digits in $15n$ are 0 or 8.

Answer: 592

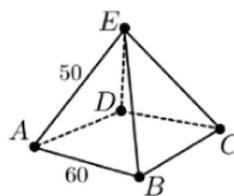
19. Define x as follows:

$$x = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}$$

Find the sum of the numerator and denominator of x when x is in the most reduced form.

Answer: 19

20. Consider a pyramid whose faces consist of a 60×60 square base ABCD and four $60 - 50 - 50$ triangles that join at the apex E . If you are only allowed to move on the triangles, what is the length of the shortest path between A and C ?



Answer: 96

21. For a positive integer n , we define $c(n)$ to be the number of ways in which we can write n as the sum of two or more positive integers. For example,

$$21 = 11 + 10 = 8 + 7 + 6 = 6 + 5 + 4 + 3 + 2 + 1$$

so $c(21) = 4$ as there are no other ways that 21 can be written as needed. What is $c(45)$?

Answer: 6

22. In a Pythagorean Triplet a, b, c such that $a < b < c$, the sum of a & c is 361 and $b = 209$. Find the value of c .

Answer: 241

23. In a sequence of integers, each number is the sum of the previous two numbers. The 8^{th} number in the sequence is 29, and the 9^{th} number is 47. What is the 5^{th} number?

Answer: 7

24. In how many ways can 10 identical candies be distributed among 4 children such that each child gets at least 1 candy, and at most 5 candies?

Answer: 68

25. The sum of the first 20 terms of an arithmetic progression is 310, and the sum of the next 20 terms is 3910. What is the common difference?

Answer: 9

26. There are 2023 numbers such that if we subtract each of them from their sum, we get back the original set of numbers. Find their product.

Answer: 0

27. Let a, b, c be positive rational numbers such that

$$a + \frac{1}{b} = \frac{7}{3}$$

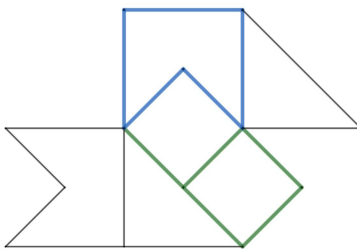
$$b + \frac{1}{c} = 4$$

$$c + \frac{1}{a} = 1$$

Find the value of abc ?

Answer: 1

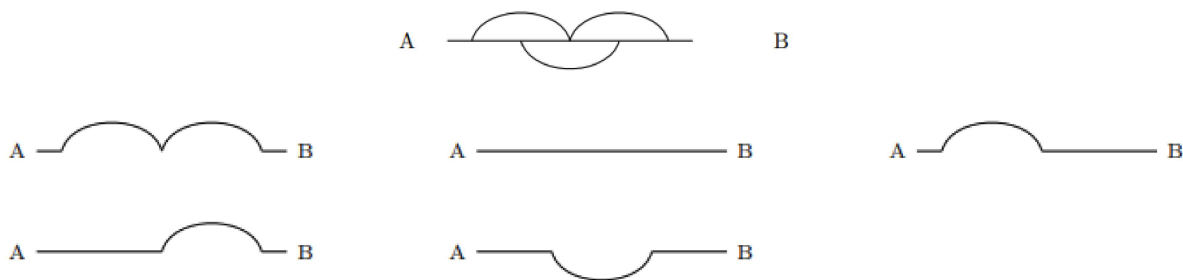
28. If the side of the green square is 2 units, find the area of the blue pentagon.



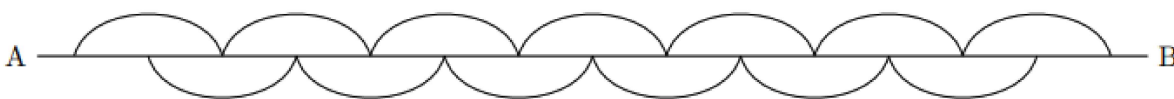
Note that the two squares, the two pentagons and the two isosceles right angled triangles are congruent respectively.

Answer: 6

29. In the diagram below, there are five left-to-right routes from A to B :



How many left-to-right routes are there from A to B in the diagram below?



Answer: 610

30. A hunter and an invisible rabbit play a game on the number line. First, the rabbit chooses on an integer between 1 and 100 (both inclusive) which is unknown to the hunter. Each day thereafter, the hunter will guess the position of the rabbit. If he is wrong, the rabbit then hops to the next integer to the right; unless it is at 100, then it stays in place. What is the least number of days the hunter needs to guarantee to be able to catch the rabbit?

Answer: 51

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