

Guess the Card

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One Saturday, during our weekly class of the All-Girls Math Nurture Camp conducted by *Raising A Mathematician Foundation*, our teacher Mr. Vinay Nair gave us an interesting problem based on a card trick.

A magician and his assistant perform the trick. Someone from the audience picks at random any five cards from a complete deck of playing cards (no Jokers). The remaining cards are kept aside. The selected five cards are handed over to the assistant, without the magician seeing which cards have been chosen. From these five cards, the assistant selects one card and gives the remaining four cards to the magician, one by one. The assistant decides the order in which to give the four cards. By looking at the four cards, the magician works out the identity of the fifth card. There is no sign language for communication between the magician and the assistant; the only communication between them lies in the way the cards are handed over. The magician finds the unknown card simply by looking at the four cards.

Swasti, Tara, Akshita, Srividya and I took up the challenge to find how it could be done.

Swasti guessed that the first of the four cards handed to the magician would indicate the suit of the unknown card. There must be at least one suit with at least two cards in a set of five cards. This can be proven by the Pigeonhole Principle, as there are 5 cards ('pigeons') and 4 suits ('pigeonholes'). The suit now being known, the number on the card remains to be found.

Tara suggested that the number may be found from the way the cards were given to the magician by the assistant — face up (FU) or face down (FD). This is a pre-decided code between the assistant and the magician. Continuing with Swasti's idea, the assistant and magician decide that the suit of the unknown card must be the suit of the first card handed over to the magician. Thus, when the magician receives the first card, he knows that

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the unknown card is among the remaining twelve cards in the suit. According to their code, if the assistant hands over the first card as face up, the unknown card must be one of the first 6 numbers and if it is face down, it must be one amongst the last 6. If the second card is handed over as face up, the unknown card must be in the first 3 cards of the suit; if it is face down, the unknown card must be among the last 3 in the suit. If the third card is face up, it must be the centre card and if it is face down, it must be one of the remaining 2 cards. If it is among the remaining 2 cards, it can be determined by the fourth card. If it is face up, it is the lower card and if it is face down, it is the higher card. For instance, if the suit is spade and the first card is 3 and the unknown card is 7, then the assistant would give the first card as face up indicating that the unknown card is amongst Ace, 2, 4, 5, 6 or 7. Since the unknown card is amongst the last three in this set (5, 6 and 7), the assistant will give the second card as face down to indicate this. As the unknown card is one of the two remaining cards, that is, 5 and 7, the assistant will give the third card face down. The assistant must give the fourth card face down to indicate that the unknown card is the higher of these, that is, the 7 of Spades. Voila! Mathemagic!

Akshita suggested that one might use the spacing between the cards when they are placed on the table while handing them over to the magician, rather than handing over the cards face up or down. This is more subtle than placing cards face up or face down. However, the magician and assistant must take precautions to ensure that they can clearly distinguish the spacing.

Srividya came up with a solution by which the card could be found by looking at just the first 3 cards. Furthermore, it makes use of four ways to give a card, i.e., Vertical Face Up (VFU), Vertical Face Down (VFD), Horizontal Face Up (HFU) and Horizontal Face Down (HFD). Horizontal and vertical refer to the direction of the orientation of the card.

The magician knows that the unknown card is from the same suit as the first card. Thus, after seeing the first card, there are 12 numbers left

for the unknown card. We rank those numbers in ascending order as 1 to 12. If the first card is given as HFU, it means the unknown card is the first amongst those ranks. If the first card is given as HFD, then it means that the unknown card is the second amongst those 12 ranks, and so on as follows:

- First Card:
 - HFU = 1st
 - HFD = 2nd
 - VFU = 3rd
 - VFD = 5th to 12th
- First and second card together = 4th
- Second Card:
 - HFU = 5th
 - HFD = 6th
 - VFU = 7th
 - VFD = 9th to 12th
- Second and third card together = 8th
- Third Card:
 - HFU = 9th
 - HFD = 10th
 - VFU = 11th
 - VFD = 12th

The rules in the above procedure require the magician and the assistant to remember quite a few things, but it helps the magician guess the unknown card with just three cards.

I presented a solution which, like Srividya's, uses four ways to give a card. The first card, as in all other solutions, shows the suit of the unknown card. The value is determined as shown below:

- First Card:
 - HFU = Ace
 - HFD = 2
 - VFU = 3
 - VFD = 4 and above;
- Second Card:
 - HFU = 4

HFD = 5
VFU = 6
VFD = 7 and above;

- Third Card:

HFU = 7
HFD = 8
VFU = 9
VFD = 10 and above;

- Fourth Card:

HFU = 10
HFD = Jack
VFU = Queen
VFD = King

Whenever a card is determined, all cards received after that are irrelevant.

However, none of these were what Vinay Sir had in mind. His solution was different, he said, and the session ended with a tantalizing hint that his solution was related to Permutations.

After the class, I sent the following solution to him and Vinay Sir confirmed that this was what he had in mind.

The first card shows the suit of the unknown card. Also, if the first card is face up, the number is one of the first 6 and if it is face down, it is among the last 6. From the remaining three cards, let the highest be a , the second be b , and the last be c , where ace is the smallest and king is the highest. If two cards have the same number, the order of suits from lowest to greatest is Hearts, Clubs, Diamonds, and Spades. The three cards may be arranged in 6 distinct orders viz. abc , acb , bac , bca , cab , and cba . Each order indicates the 1st, 2nd, 3rd, 4th, 5th, and 6th numbers respectively.

So, we arrived at 4 distinct solutions for how the magician could have guessed the unknown card based on the way the assistant arranged the remaining cards.

On further thinking, I realized that it is possible to do the same trick by picking four cards instead of five where only three cards indicate the fourth (unknown) card.

The first card indicates the suit. However, since there are only four cards, all four may possibly be of different suits. Therefore, the color of the first card is the same as that of the unknown card. If the first card is face up, it is of the same suit. If it is face down, it is of the other suit (of the same color).

The second card and third card indicate the number on the unknown card.

Each card can be placed in 4 ways — Horizontal Face Up (HFU), Horizontal Face Down (HFD), Vertical Face Up (VFU) and Vertical Face Down (VFD).

The second card indicates the number on the card mod 4. Let HFU, HFD, VFU and VFD stand for 0, 1, 2, and 3 respectively. For the third card, let HFU, HFD, VFU and VFD stand for x , $x + 4$, $x + 8$ and $x + 12$ respectively, where x is the value derived by the second card. Thus, if the second card indicates 2 and the third card indicates $x + 4$, the number on the unknown card is 6. The suit is decided by the first card.

We have therefore established that one does not require 4 cards to guess the unknown card. It can be done with 3.

Can it be done with 2 cards?

I don't know... yet.

I'm working on it. If you get it before me, let me know!



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