Raising A Mathematician Foundation®

Roots for Teachers 2025

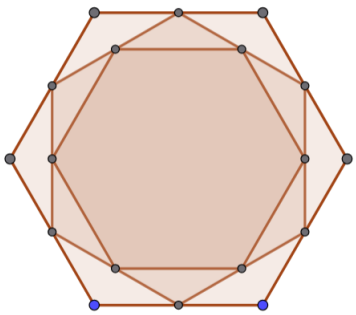
Roots for Teachers 2025

October 13, 2025

Questions

1. If
$$\frac{1}{1\times 3}+\frac{1}{2\times 4}+\frac{1}{3\times 5}+\cdots+\frac{1}{2025\times 2027}=\frac{a}{b}$$
where $\gcd(a,b)=1$, find the remainder when $a+b$ is divided by 9.
Answer: 1

2. All the hexagons in the following figure are regular. If a/b is the ratio between the areas of the biggest and the smallest hexagon, where $\gcd(a,b)=1$, find $a+b$.



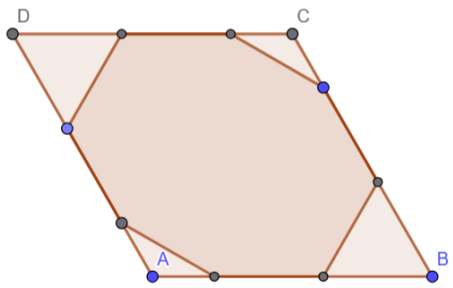
Answer: 25

3. The sum of the number of diagonals in three distinct regular polygons is 38. Find the sum of the number of sides.
Answer: 19

4. Find the number of zeroes at the end of $3\times 6\times 9\times 12\times\cdots\times 2025$ in base 5.
Answer: 168

5. A cube of side 21 has a sphere inscribed inside it, which has a cube inscribed inside it which has another sphere followed by a cube inscribed in that order. Find the volume of the smallest cube in this.
Answer: 343 .

6. Given a rhombus $ABCD$ with $\angle ABC = 60^\circ$ and $AB = 6$ units. An octagon with all sides equal is constructed so that both the diagonals act as axes of symmetry. Let the side of the octagon be $\frac{a+b\sqrt{3}}{c}$. What is $a + b + c$ where a, b, c are integers with $\gcd(a, c) = \gcd(b, c) = 1$?



Answer: 41

7. Given two numbers with their sum of fourth powers as 1552 and product 24, find the absolute value of their sum.
Answer: 10

8. The difference between the sum of the first n squares and the sum of the first n positive integers is 1120. What is n?
Answer: 15

9. How many integers x are there such that $(2x - 1^2)(2x - 2^2)(2x - 3^2)(2x - 4^2)(2x - 5^2) \cdots (2x - 29^2)(2x - 30^2)$ is negative?
Answer: 225

10. If
$$y = \frac{x^2 - x + 1}{x^2 + x + 1},$$
where x is a real number, find the maximum value of y.
Answer: 3

11. The last two digits of the 200 digit number 12301230012300012300001 . . . are?
Answer: 00

12. In how many ways can you choose three distinct numbers from the set $\{1, 2, 3, \ldots, 20\}$ such that their product is divisible by 4?
Answer: 795

13. N is a 50 digit number. All the digits except the 26th digit from the right are 1. If N is divisible by 13, then find the unknown digit.
Answer: 9

14. The coefficients of three consecutive terms in the expansion of $(1+t)^n$ are $120, 210, 252$. Then n must be?
Answer: 10

15. An astronaut lands on a planet and meets a native of the planet. She asks the native "How many days do you have in your year?" He answers "It is the sum of squares of three consecutive natural numbers but it is also the sum of squares of the next two numbers." The answer to the astronaut's question is?
Answer: 365

16. The letters of the word "MOTHER" are permuted and all the permutations so formed are arranged in alphabetical order as in a dictionary. Then the number of permutations which come before the word "MOTHER" is?
Answer: 308

17. One plant is now 44 cm tall and will grow at the rate of 3 cm every two years. A second plant is now 80 cm tall and will grow at a rate of 5 cm every six years. In how many years will the plants be the same height?
Answer: 54

18. There are 11 points on a plane with 5 lying on one straight line and another 5 lying on a second straight line which is parallel to the first line. The left over point is not collinear with any two of the previous 10 points. What is the number of triangles that can be formed using vertices chosen from these 11 points?
Answer: 145

19. Suppose that $F(n+1) = \frac{2F(n)+1}{2}$ for $n = 1, 2, 3, \ldots$ and $F(1) = 2$. Find the value of $F(101)$.
Answer: 52

20. Evaluate $1^2 + 2^2 - 3^2 - 4^2 + 5^2 + 6^2 - 7^2 - 8^2 + \cdots - 1000^2 + 1001^2$.
Answer: 1001

21. Find the least integer n greater than 345 such that $\frac{3n+4}{5}$, $\frac{4n+5}{3}$ and $\frac{5n+3}{4}$ are all integers.
Answer: 397

22. Suppose that f is a function such that

$$3f(x) - 5xf\left(\frac{1}{x}\right) = x - 7$$

for all non-zero real numbers x. Find $f(2025)$.
Answer: 4051

23. Find the positive integer n such that n^2 is the perfect square closest to $8 + 16 + 24 + \cdots + 8040$.
Answer: 2011

24. If a person's ages in 2024 and 2025 are divisors of his birth year, find the sum of all possible birth years.
Answer: 8038

25. A sailor claims he saw a dolphin while sailing this morning.

    - His friend says that the probability of spotting a dolphin on any given morning is $\frac{1}{10}$.
    - Another friend says that 3 out of 4 times, the sailor lies about what he saw.

    Given this information, what is the reciprocal of probability that the sailor actually saw a dolphin this morning?
Answer: 28

26. In a world where everyone wants a girl child, each family continues having babies till they have a girl, and they will stop once they have a girl child. What is the expected ratio of boys to girls in this town? *(Assume that the probability of having a boy or a girl is the same.)*

    (a) 2:1

    (b) 1:1

    (c) Cannot be determined

    (d) None of these

    Answer: 2 or (b)

27. Let α and β be the roots of the equation $2x^2 - 4x - 6 = 0$. Let $A_n = \alpha^n + \beta^n$. Find

$$\frac{A_{100} - A_{98}}{A_{99} + A_{98}}.$$

    Answer: 2

28. A boy comes from a family of 2 children. What is the reciprocal of probability that the other child is his older sister? (Ignore the possibility of twins.)
Answer: 4

29. The length of a common internal tangent to two circles is 5 and a common external tangent is 15, then the product of the two radii is?
Answer: 50

30. The sum of an infinite geometric progression (G.P.) is 4. The second term of this G.P. is x. If $x \in (a, b] \setminus \{c\}$, then $-a + b + c =$?
(Note: $(m, n]$ represents the set of all x such that $m < x \leq n$)
(Note: The "\" symbol represents the set difference. $A \setminus B$ represents the set of all elements that are in A, but not in B.)

Answer: 9