

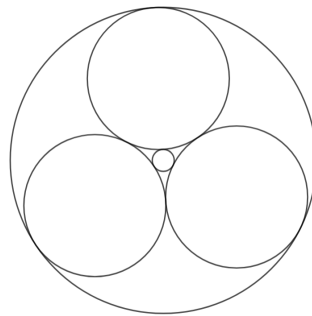
## ROOTS 2025

Square Roots

September 22, 2025

### Answer

1. Given a sphere of 25 units radius with center  $O$ . A circle of diameter 40 units is drawn on its surface. Let  $A$  be the center of the circle (which lies on the same plane as the circle). Calculate the distance  $OA$ .  
**Answer:** 15
2. How many integer values exist for  $x$  such that  $(2x - 1)(2x - 4)(2x - 9)(2x - 16)(2x - 25)(2x - 36)$  is negative?  
**Answer:** 9
3. Find the number of 3 digit palindromes that are divisible by 7. (Note: Palindromes are numbers like 717 which look the same when read from left-to-right or right-to-left)  
**Answer:** 12
4. A person wanted to cover a certain distance in a specific amount of time. However, they covered the first half of the path with only  $(\frac{2}{3})^{\text{rd}}$  of the required speed. If they should increase speed to  $k$ -times the current speed to reach in time, find  $k$ .  
**Answer:** 3
5. If the product of the radii of the biggest and the smallest circle in the following figure is  $a/b$  where  $\text{gcd}(a, b) = 1$ , and the 3 identical circles have radius 1, find the value of  $a + b$ .



**Answer:** 4

6. In the sequence 2025, 20252025, 2025202520252025,  $\dots$  every subsequent term contains twice as many digits as the previous one. Find the remainder when the 2025<sup>th</sup> term in this sequence is divided by 7.  
**Answer:** 2
7. If the product of 2 natural numbers is 6 more than their sum, find the square of the larger number.  
**Answer:** 64

8. Find the value of  $16(x^x + (-x)^{-x})^x$  for  $x = 2$ .

**Answer:** 289

9. If the difference between squares of 2 natural numbers is 197, find their sum.

**Answer:** 197

10. Find the ratio of volumes of a cylinder and a sphere, if the sphere takes half the volume compared to a cone with half the height and twice the radius of the cylinder.

**Answer:** 3

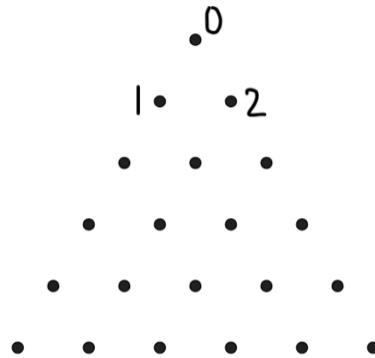
11. If

$$\sqrt{\frac{x}{1^3}} + \sqrt{\frac{x}{1^3 + 2^3}} + \sqrt{\frac{x}{1^3 + 2^3 + 3^3}} + \dots + \sqrt{\frac{x}{1^3 + 2^3 + \dots + 21^3}} = 21,$$

find  $x$ .

**Answer:** 121

12. Every number on the following grid is filled with 0, 1, 2 so that the sum of three numbers on the vertices of any equilateral triangle of side length one is exactly 3. What is the sum of numbers in the 2025<sup>th</sup> row?



**Answer:** 2025

13. L.C.M. of two numbers is 2277. Their H.C.F is a perfect square greater than 1. Find the least possible value of their sum.

**Answer:** 306

14. Person A invested \$20000 in a bank for compound interest of 8% per annum. How much should Person B invest so that their capital is same as Person A after 4 years if they receive a compound interest rate of 20% per annum?

**Answer:** 13122

15. The area of an equilateral triangle inscribed in a circle is  $\frac{k}{4\pi}$  times the area of the circle. Find  $k^2$ .

**Answer:** 27

16.

$$x = 3 + \frac{4}{3 + \frac{4}{3 + \frac{4}{\dots}}}$$

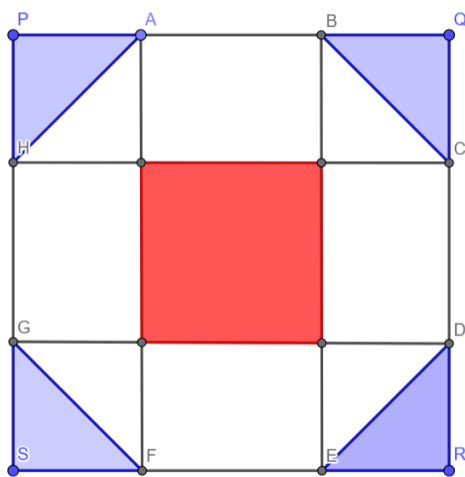
Find  $x^x$ .

**Answer:** 256

17. Find the remainder when  $2^{1224}$  is divided by  $2^{408} - 2^{204} + 1$ .

**Answer:** 1

18. A bottle of water is shaped like a cylinder with a cone on its top. The circular base is of radius 5 units, the height of the cylinder is 10 units and the height of the cone is 6 units. At what height is the bottle half full?  
**Answer:** 6
19. A city is divided into four regions. The city council has decided that a new city hall, a new school, and a new movie theatre shall be built. The only condition is that the school and the movie theatre must not be in the same region. How many ways can these four buildings be built in the city?  
**Answer:** 48
20. In eight years Ravi will be three times the age of that Rupali was last year. Twenty five years ago their ages added to 83. How old is Ravi now?  
**Answer:** 97
21. As described in the figure below, an octagon  $ABCDEFGH$  with all sides equal is constructed inside of a square  $PQRS$ . The length  $PQ$  is given to be 24. If  $A$  is the total violet area, and  $B$  is the total red area, find  $A - B$ .



**Answer:** 0

22. The digits 1, 2, 3, and 4 can be arranged in 24 different four digit numbers. What is the sum of those 24 numbers?  
**Answer:** 66660
23. How many different positive integers divide  $10!$  ? (Note:  $10!$  or 10 factorial is the product of first 10 positive integers)  
**Answer:** 270
24. Evaluate the product  $(1 + \frac{2}{3})(1 + \frac{2}{4})(1 + \frac{2}{5})(1 + \frac{2}{6}) \dots (1 + \frac{2}{98})$ .  
**Answer:** 825
25. In a factory, the cost of producing each item is inversely proportional to the square root of the number of items is produced. The cost of producing ten items is Rs. 2100. If items sell for Rs. 30 each, how many items need to be sold so that the producers have no profit and no loss.  
**Answer:** 490
26. The set  $S$  contains nine numbers. The mean of the numbers in  $S$  is 202. The mean of the five smallest of the numbers in  $S$  is 100. The mean of the five largest numbers in  $S$  is 300. What is the median of the numbers in  $S$ ?  
 (Note: Mean of numbers is the ratio of their sum to the number of numbers. For example, if we have 5 numbers  $a, b, c, d, e$ , their mean will be  $\frac{a+b+c+d+e}{5}$ )

(Note: Median of numbers is the middle term when the numbers are arranged in an ascending order. Eg. In the numbers 1, 3, 15, 2, 12, The number 3 will be the median)

**Answer:** 182

27. A man met a tortoise under a tree. When the tortoise was the man's age, the man was only quarter his current age. When the tree was the tortoise's age, the tortoise was only a seventh of tortoise's current age. If the sum of their current ages is now 264, how old is the man?

**Answer:** 44

28. A train car held 6000 kg of mud which has 88% water. Under the sun, some of the water evaporated bringing down the mud to have 82% water. What is the weight of this mud with 82% water?

**Answer:** 4000

29. Let  $a_1 = 2$ , and for  $n \leq 1$ , let  $a_{n+1} = 2a_n + 1$ . Find the smallest value of an  $a_n$  that is not a prime number.

**Answer:** 95

30.  $\frac{a+3}{a-3} = \frac{b+4}{b-4} = \frac{c+5}{c-5} = \frac{d+6}{d-6}$ . Find  $\frac{a+d}{b+c}$ .

**Answer:** 1