Online Contest for Grade 7 and 8

THE DEEPER YOU GO, THE MORE YOU GROW!

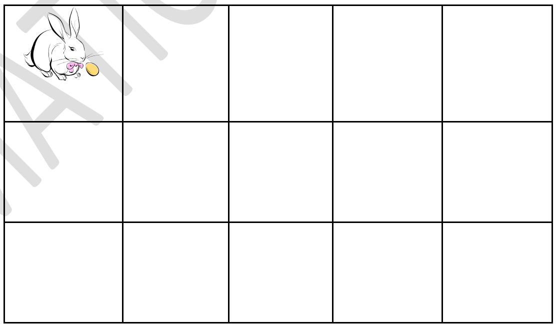
RAISING A MATHEMATICIAN FOUNDATION

Solutions for SQUARE ROOTS

1)

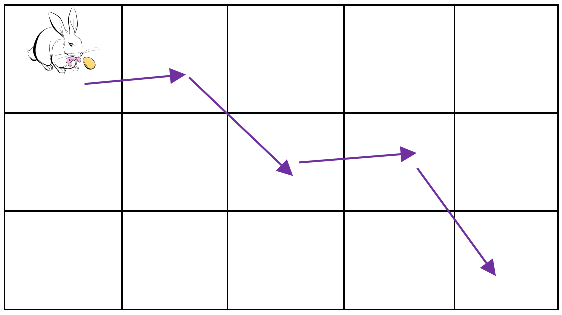
In one move, the rabbit in the top leftmost corner of the 3×5 grid can go to any adjacent square along a row, column, or diagonal.

The minimum number of moves that can be made by the rabbit to move to the bottom rightmost corner is 4.

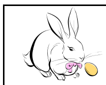


Here is one such example.

In how many ways the rabbit can make 4 moves to move from the top leftmost corner of the 3×5 grid to the bottom rightmost corner? ________.



Solution:

	1	2	0	0
0	1	2	5	0
0	0	1	3	8

Observe that we can mark each cell with the number of paths by which the rabbit can reach it enroute to the destined cell (bottom most corner).

∴ The number of paths of reaching the destined cell in 4 moves by the rabbit is 8.

Note: The rabbit should not travel within the same column otherwise it cannot reach the destined cell in exactly 4 moves.

Answer: 8

2) Teacher asked her students to collect as many 4-digit numbers satisfying the following condition: Each 4-digit number should be of the form:

Place Value:	*Thousands*	*Hundreds*	*Tens*	*Units*
Number	*Number of '1's in the number*	*Number of '3's in the number*	*Number of '2's in the number*	*Any valid digit*

After some time, Shreya collected only 2 such 4-digit numbers: 2113 and 1022.

Swetha collected all such numbers.

The number of 4-digit numbers collected by Swetha is _____.

Solution: Note that except the units place 3 cannot occur in any other place.

We can have either 1 or 2 in thousands place. If we have 2, then we need to place two 1's and one 3 or two 1's and one 0. We get the numbers 2113 and 2011.

If we have 1 in thousands place, then the condition in thousands place is satisfied. We cannot use a 3 as it will lead to one more 1 in hundreds place.

So, we can have 0's and 2's only, leading to 1022.

What about unit's place? We can have any digit > 3 there and it will not contribute to the count of any position.

We can get 1000, 1004, 1005,1006, 1007, 1008, 1009 as valid numbers.

No other number is possible. So only 10 numbers.

Answer: 10

3) ΔABC is a triangle with side AC = 25 unit, AB = c unit and BC = a unit, where a, c are positive integers. Neither AB nor BC is longer than AC.

Also, $c + z = a + x$ where z and x are the length of the altitudes falling on AB, BC respectively. How many solutions of the ordered pair (c, a) are there? __.

Solution: Area of $\Delta ABC = \frac{1}{2}.c.z = \frac{1}{2}.a.x$ and $c + z = a + x$ (given)

$\Rightarrow c - z = a - x \quad (or) \quad c - z = x - a \because (c - z)^2 = (c + z)^2 - 4cz$

Case (i): Isosceles triangle with $c = a$ and $z = x$. Possible values for $c = a$ are 13,14, 15, …, 24, 25. There are 13 such triangles.

In these the altitudes $z = x$ need not be integers.

Case(ii): Non-isosceles triangles with $c = x \text{ and } z = a$.

If either one of z or x is an integer the other is also an integer.

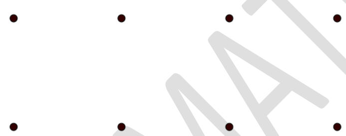
The possibilities are a is the altitude on c and c the altitude on a.

This is possible only for right angled triangles with integer sides and AC as hypotenuse. The triangles are the triples (25, 20, 15) & (25, 24, 7).

This gives the (c, a) pairs as (20, 15), (15, 20), (24, 7), (7, 24).

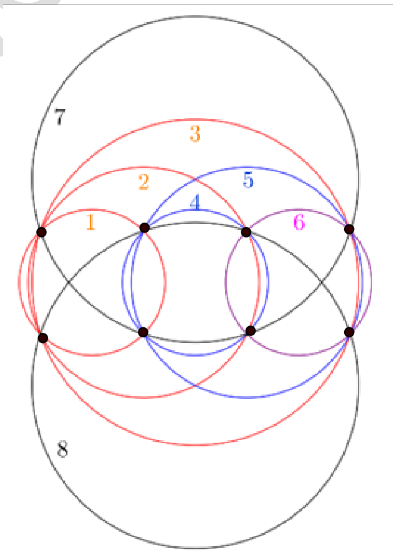
Totally we have 17 ordered pairs. **Answer: 17**

4) Here you observe a 2 x 4 dot grid (2 rows of 4 dots in each row)



How many different circles can be drawn that passes through *exactly* 4 points among 8 dots of this grid? _____.

Solution:



Take pairs of columns from these four columns of dots. Each will have a circle going through the four points. This gives you six circles. In addition, look at the two end points in the first row and two middle points in the second row. The four points have a circle going through them. Also, look at the two end points in the second row and two middle points in the first row. These four points also have a circle going through them. Thus, there are eight circles going through exactly four points of the grid. **Answer: 8**

5) One of the sides of a triangle is 4 *cm* and the length of the median falling on it is 1 *cm*. Which of the following is definitely true?

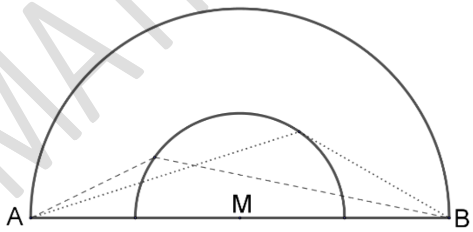
A) The triangle is an acute angled triangle.

B) The triangle is an obtuse angled triangle.

C) The triangle is a right-angled triangle.

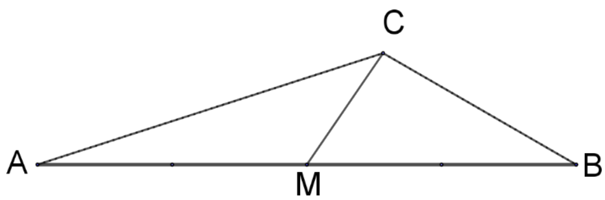
D) The triangle is a scalene triangle.

Solution 1:



Take a 4 cm line segment AB with midpoint M. Draw a circle with M as centre and radius 2. Draw a circle with centre M and radius 1. This is entirely within the first circle. The vertex C of the required triangle must lie on the inner circle as the median length is 1. As C lies within the circle, $\angle ACB$ is obtuse. Here we use the fact that the angle subtended by a diameter in a semi-circle is 90^o and at any interior point is $> 90^o$.

Solution 2:



AM = MB = 2 and MC =1. Therefore, from triangles AMC and BMC we get $\angle MAC < \angle MCA$ and $\angle MBC < \angle MCB$.

Hence, $\angle A + \angle B < \angle MCA + \angle MCB = \angle C$.

Hence, $180^o = \angle A + \angle B + \angle C < 2\ \angle C$. Therefore $\angle C > 90^o$

Answer: B

6) How many 2-digit numbers exist such that the product of the digits of the number is exactly divisible by the sum of the digits of the number?

A) 10 B) 12 C) 15 D) 16

Solution: Consider the 2-digit numbers with 0 as units place. There are 9 of them and they satisfy the required condition as y = 0 and x is non-zero. Look at the 2-digit numbers with repeated even digits 22, 44, 66, 88. Here too the product y is divisible by the sum x. Then we have two numbers with distinct digits namely 36 and 63 where x = 9 and y = 18. There are no more numbers. Hence, we have 9 + 4 + 2 = 15 such numbers.

Answer: C

7) 10 students together collected 250 marbles. No two collected same number of marbles. No one collected prime number of marbles. Each collected odd number of marbles. If student *Vinay* collected the largest number of marbles, how many marbles were collected by *Vinay*? ______.

Solution:

List of prime numbers is 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, … …

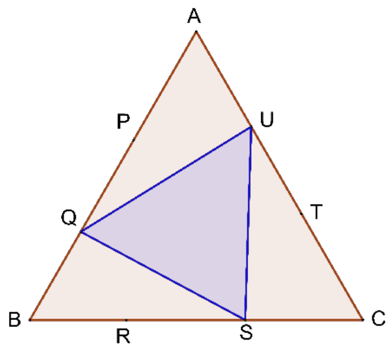
The least non-prime odd number is 1 and the next is 9.

Look at the odd composites 15, 21, 25, 27, 33, 35, 39, 45. Sum of these is 250.

Hence Vinay collected 45 marbles.

Answer: 45

8) Here you observe an equilateral triangle Δ ABC. P, Q are trisection points of AB; R, S are trisection points of BC; T, U are trisection points of CA, as shown. What fraction of △ABC is the shaded △UQS?

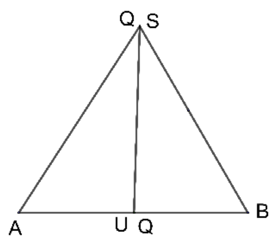


A) $\frac{3}{10}$ B) $\frac{4}{9}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$

Solution 1: Note that the three triangles AQU, BSQ and CUS are congruent since ABC is equilateral. The altitude from U to BC will be two-thirds the altitude from A to BC. Area of Δ CUS = $\frac{1}{2}$ × SC × altitude from U to BC = $\frac{1}{2} \times \frac{1}{3}$ BC× $\frac{2}{3}$ altitude from A to BC = $\frac{2}{9}$ area of Δ ABC. Total area of the three triangles AQU, BSQ and CUS is $\frac{6}{9}$ area of Δ ABC= $\frac{2}{3}$ area of Δ ABC.

Hence area of Δ CUS = $\frac{1}{3}$ area of Δ ABC.

Solution 2:



If we join the triangles AQU and BSQ along QU = SQ, i.e., glue the sides QU with SQ we get an equilateral triangle with angles A and B and side two-thirds the original triangle. Hence area will be $\frac{2}{3} \times \frac{2}{3}$ area of Δ ABC.

Hence each triangle AQU, BSQ and CUS will have $\frac{2}{9}$ area of Δ ABC.

Hence area of Δ CUS = $\frac{1}{3}$ area of Δ ABC. **Answer: C**

9) A finite sequence has all the 10-digit numbers with different digits, written in ascending order as follows: 1023456789, 1023456798, 1023456879, …, 9876543201, 9876543210. It has a total of 3265920 terms in it.
It's 100^{th} term will be ________________..

Solution: Look at any 4 distinct digits a < b < c < d. We can get 24 numbers re-arranging these and they can be put in increasing order. Similarly, we can get 6 with three digits a < b < c.
To start with interchange 8 and 9 to get the 2^{nd} number. Now place 8 in the eight position and get 2 more and 9 in the eighth position and two more. In all these 6 numbers the first 7 digits were not disturbed.
Move 7 to the seventh position and arrange 6, 8, 9 in 6 ways and put them in increasing order. Similarly with 8 and 9 in seventh from left, position respectively. We have 24 numbers now. Successively move 6, 7, 8 to the 6^{th} position from left, without disturbing the first five positions, rearrange the rest of the digits in 4 digit numbers in increasing order. These give 24 + 24 + 24 numbers. We have covered 96 numbers. So now 9 is in 6^{th} position and 5, 6, 7, 8 are to be arranged in 4-digit numbers in increasing order. They are 5678, 5687, 5768, 5786. Hence the 100^{th} number in the sequence is 1023495786.

Answer: 1023495786

10) A lift in "*Roots*" Apartments takes 5 seconds for a movement from a floor to its immediate next floor either up or down. "*Roots*" Apartments has 3 floors above the ground floor.
On a given day, the lift started operating from ground floor at *8:00 AM.*
After a few movements of up and down, the lift has reached the topmost (3rd) floor just at the time of *5 seconds past 8:01 AM* on the same day.
During this interval time, the lift has halted at ground floor, 1st floor and the 2nd floor exactly once (not in this order), 10 seconds being the halted time in each floor. Where will be the lift exactly at 56 seconds past 8:00 AM?

A) halted at 2nd floor B) halted at 1st floor
C) moving up between 1st and 2nd floor D) moving down between 2nd and 1st floor

Solution:

The net duration for the travel of the lift from 8:00 am to 5 seconds past 8:01 am is 65 seconds. From the given statements, the lift has halted at three floors:

Ground Floor, First Floor and Second Floor in some order.

The total halted duration is 30 seconds.

The remaining time is 35 seconds where the lift will be moving and covering 7 floors in total (that should repeat - crossed floors).

Therefore, the movement of the lift has the following two possibilities:

1) $GF \rightarrow 1F \rightarrow 2F \rightarrow GF \rightarrow 3F$
2) $GF \rightarrow 2F \rightarrow 1F \rightarrow GF \rightarrow 3F$

In either of the two cases, the lift will be between 1F and 2F @ 56 sec past 8:00 am.

Answer: C

11) A sequence has 99 fractions, all are of different values. Each one shares a common property that the sum of the numerator and denominator is 100.

The sequence is:

$$\frac{1}{99}, \frac{2}{98}, \frac{3}{97}, \frac{4}{96}, \frac{5}{95}, \ldots, \frac{97}{3}, \frac{98}{2}, \frac{99}{1}.$$

How many of the fractions among these 99 fractions give integer values?

A) 10 B) 8 C) 12 D) 9

Solution: If the denominator is an exact divisor of the numerator, then it must be a divisor of the sum 100. Conversely if n is a divisor of 100 then, $\frac{100-n}{n}$ is an integer. Hence, we need to count the divisors of $100 = 2^2 \times 5^2$. The divisors are 1,2,4,5,10,20, 25, 50. 100 is not counted for obvious reasons. Total is 8.

Answer: B

12) a, b, c, d, e, f are six different natural numbers. What will be the maximum possible number of primes among the 15 sums:

$$(a+b),(a+c),(a+d),(a+e),(a+f),(b+c),(b+d),(b+e),$$

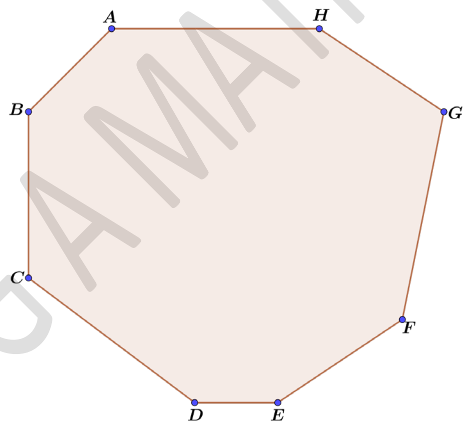
$$(b+f),(c+d),(c+e),(c+f),(d+e),(d+f),(e+f)?$$ ______.

Solution: To maximise the number of primes, we must look at odd primes. To get an odd sum one term must be odd and the other even. Hence with three odd numbers and three even numbers among the six we can get 9 odd sums. Can they be chosen to be primes? Yes!

Look at $a = 2, b = 4, c = 8$ and $d = 3, e = 9, f = 15$. We get the sums $(a+d),(a+e),(a+f),(b+d),(b+e),(b+f),(c+d),(c+e),(c+f)$ to be $5, 11, 17, 7, 13, 19, 11, 17, 23$ all nine prime numbers!

Answer: 9

13) Here you observe a convex octagon.



A diagonal is chosen so that the number of diagonals intersecting the chosen diagonal in the interior region of the octagon is 9.

How many possibilities are there for the chosen diagonal?

A) 4 B) 6 C) 8 D) 0

Solution: For a diagonal to cross the chosen diagonal the endpoints of the diagonal must be on opposite sides of the chosen diagonal. Hence, to get 9 intersecting diagonals, there must be three vertices each, on either side of the chosen diagonal.

This means the endpoints of the chosen diagonal must have three vertices between them. There are 4 such diagonals AE, BF, CG, DH.

Answer: A

14) In how many ways can you trace out ROOTS in this diagram if you are only allowed to move one step at a time horizontally or vertically, up or down, forward or backward? _____.

```
        S
      S T S
    S T O T S
  S T O O O T S
S T O O R O O T S
  S T O O O T S
    S T O T S
      S T S
        S
```

Solution:

To reach the destination S, we must reach T immediate before to S.

The number of ways of reaching each T in the leftmost, topmost, rightmost or bottommost is 1. Each of other T's, we can reach in 3 possible ways.

Therefore. number of ways of reaching each S in the leftmost, topmost, rightmost or bottommost is 1 through the respective T's.

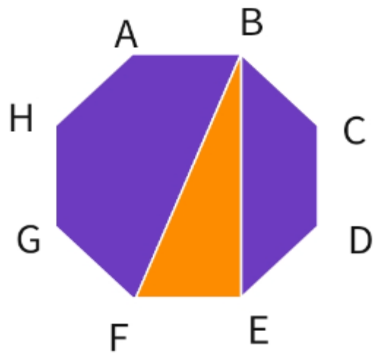
Each of the middle S's in the boundary can be reached in 3+3=6 ways through respective T's. Each of all other S's in the boundary can be reached in 1+3=4 ways through respective T's.

Therefore, the number of ways of tracing out ROOTS in this diagram equals the number of ways of reaching one of the S's in the boundary through respective T's, which is $4 \times 1 + 4 \times 6 + 8 \times 4 = 60$.

Answer: 60

15) $ABCDEFGH$ is a regular octagon.

What fraction of the area of the octagon will the area of triangle BEF form? ___.



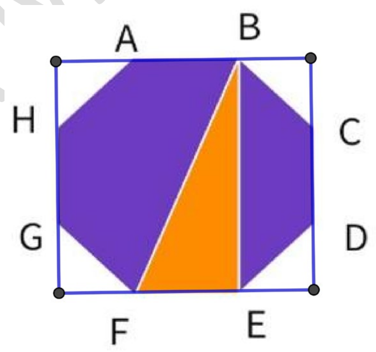
A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{2}{5}$ D) $\frac{3}{8}$

Solution: Complete a square around the regular octagon, as shown.

Let the length of each side of the regular octagon be $\sqrt{2}$ unit.

Each of the four congruent right triangles surrounding the regular octagon will have the side lengths 1 *unit*, 1 *unit* and $\sqrt{2}$ *unit*.

This is because the exterior angle of each corner of the regular octagon is $45°$.



Then, the length of the side of the square must be $(2+\sqrt{2})\ unit$.

$$Area(regular\ octagon) = (2+\sqrt{2})^2 - 4 \times \frac{1}{2} \times 1 \times 1 \quad sq.\ unit.$$

$$= 4(\sqrt{2}+1) sq.\ unit.$$

$$Area(\triangle\ BFE) = \frac{1}{2} \times \sqrt{2} \times (2+\sqrt{2}) = (\sqrt{2}+1) sq.\ unit.$$

Therefore $\frac{Area(\triangle BFE)}{Area(regular\ octagon)} = \frac{1}{4}$. **Answer: A)**

16) Here is a multiplication fact, as shown below.

$$
\begin{array}{rccccccc}
 & & & & F & I & V & E \\
 & & & \times & F & I & V & E \\
\hline
 & & & * & * & * & * & F \\
 & & * & * & * & * & I & \\
 & * & * & * & * & V & & \\
* & * & * & * & E & & & \\
\hline
* & * & * & * & * & * & * & F \\
\hline
\end{array}
$$

It shows the multiplication of a 4-digit number $FIVE$ with itself, to get an 8-digit result. F, I, V, E are different digits in order. Each * stands for a single digit not necessarily of same value. Then the value of $F - I - V + E$ is ______.

Solution: From the multiplication process, we can observe that

- $E \times E$ should give a result that ends with F.
- $E \times V$ should give a result that ends with I.
- $E \times$ I should give a result that ends with V.
- $E \times F$ should give a result that ends with E.

  E cannot be 0, 1, 5 or 6.

Exploring the possibilities, we get two possible results for $FIVE$.

They are $FIVE = 6284$ or $FIVE = 6824$.

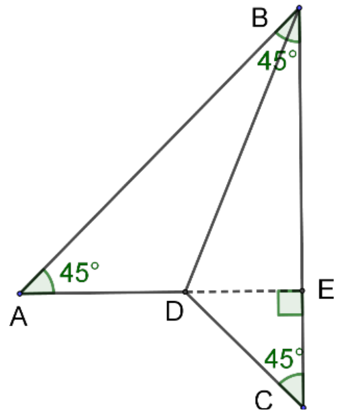
In both these possibilities, we get $F - I - V + E = 0$.

Answer: 0

17) $ABCD$ is a quadrilateral with $\angle A = \angle B = \angle C = 45^\circ$ and $\angle D = 225^\circ$.

If $BD = 5$ cm, then twice the area of $ABCD$ in $sq.\ cm$ is -------.

Solution:



Extend AD to meet BC in E. Since $\angle C = 45^{\circ}$ and $\angle CDE = 45^{\circ}$ (as $\angle ADC = 225^{\circ}$ and ADE is a straight line), $\angle CED = 90^{\circ}$.

Let $CE = DE = a$ and $BC = b$. Hence $BE = b - a = AE$.

Area of $ABCD$ = area of $\triangle ABE$ + area of $\triangle CDE$

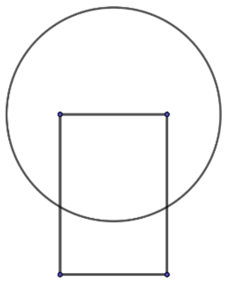
$$= \frac{1}{2} \times (b - a)^2 + \frac{1}{2} \times a^2 = \frac{1}{2} \times (BE^2 + DE^2)$$

$$= \frac{1}{2} \times BD^2 = \frac{25}{2} = 12.5 \ sq.\ cm$$

Therefore, twice the Area of $ABCD = 25$ *sq. cm*

Answer: 25

18) The circle and rectangle overlap each other. The area of the circle is four times the area of the rectangle. If the common area is removed, then the remaining area of the circle is 45 sq. cm more than the remaining area of the rectangle. Then the area of the circle is ______ *sq. cm.*



Solution:

Let the common area be x *sq. cm*, the area of the circles outside the common portion be $y\ sq.cm$ and the area of the rectangle outside the common portion be $z\ sq.cm$. It is given that $x + y = 4(x + z)$ and $y - z = 45$.

Since $z = y - 45$, we have $x + y = 4(x + y - 45)$.

This results in $3(x + y) = 4 \times 45$.

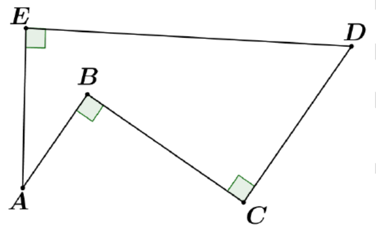
Therefore, $x + y = 60$ *sq. cm*

Answer: 60

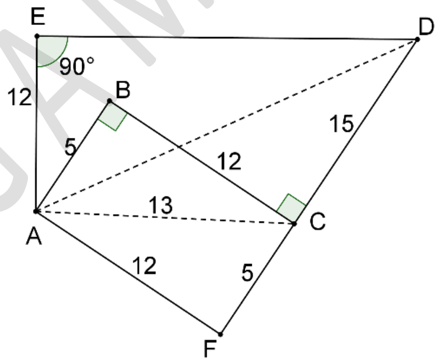
19) $ABCDE$ is a pentagon with $\angle C = \angle E = 90^\circ$ and $\angle B = 270^\circ$.

If $AB = 5$ unit, $BC = 12$ unit, $CD = 15$ unit and $AE = BC$, then $DE =$ ___unit.

A) $\sqrt{544}$ B) 20 C) $4\sqrt{30}$ D) 22



Solution:



By Pythagoras' theorem $AC = 13$ *units*. Extend DC and drop a perpendicular from A on it to meet it at F. $\triangle AFD$ is right angled at F.

In right $\triangle AFD$ and $\triangle AED$, AD is common and $AE = BC = AF$.

The two triangles are congruent.

Hence, $DE = DF = DC + CF = 15 + 5 = 20$ *units*

Answer: B

20) If the perimeter of a rectangle is 52 *cm* and the diagonals are of length 20 *cm*, then the area of the rectangle in cm^2 is --------.

Solution: Let a, b be the lengths of the sides of the rectangle in cm.

Then $a + b = 26$ cm and the diagonal = $\sqrt{a^2 + b^2} = 20$.

Hence $a^2 + b^2 = 400$. Thus, $2ab = 26^2 - 400 = 676 - 400 = 276$.

Therefore, Area of rectangle = $ab = 138$ *cm*^{2}. **Answer: 138**

21) The value of $\sqrt{51 - \sqrt{2600}} - \sqrt{51 + \sqrt{2600}}$ is

A) 10 B) –10 C) $2\sqrt{26}$ D) $10 - 2\sqrt{26}$

Solution: $\sqrt{51 - \sqrt{2600}} = \sqrt{(\sqrt{26})^2 + (\sqrt{25})^2 - 2\sqrt{26 \times 25}}$

$= \sqrt{26} - \sqrt{25} = \sqrt{26} - 5$

$\sqrt{51 + \sqrt{2600}} = \sqrt{(\sqrt{26})^2 + (\sqrt{25})^2 + 2\sqrt{26 \times 25}}$

$= \sqrt{26} + \sqrt{25} = \sqrt{26} + 5$

Hence, $\sqrt{51 - \sqrt{2600}} - \sqrt{51 + \sqrt{2600}} = -10$ **Answer: B**

22) The reverse of the number 129 is 921 and their sum **1050 is a multiple of 30.** How many three-digit numbers have the property that when added to their reverse, **the sum is divisible by 30**? _______.

Solution:

Let the three-digit number be ABC (digits in order) and its reverse be CBA.

So, we have $(100A + 10B + C) + (100C + 10B + A)$ is divisible by 30.

$101\,(A + C) + 20B$ is divisible by 30.

$A + C$ must be divisible by 10. But $0 < A + C < 19$. $\therefore A + C = 10$.

This implies that $1010 + 2B$ is divisible by 30 or $101 + 2B$ is divisible by 3.

That is, $2(1 + B)$ is divisible by 3 or $(1 + B)$ is divisible by 3.

Therefore, $B = 2, 5$ or 8 only.

The combination of values of (A, C) are

$(1,9), (2,8), (3,7), (4,6), (5,5), (6,4), (7,3), (8,2), (9,1)$.

∴ The number of such 3-digit numbers is the product of the number of possible values of B and the number of possible combinations of (A, C), which is $3 \times 9 = 27$.

Answer: 27

23) ABCD is a quadrilateral such that the perimeters of the triangles, ABC, BCD, CDA, DAB are all equal. Then the quadrilateral must be a

A) rhombus B) parallelogram but not a rhombus

C) rectangle D) none of these

Solution: From triangles ABC and BCD we get AB + AC = BD + CD.

We get AB – CD = BD – AC --------- (1)

From triangles CDA and DAB we get CD + AC = BD + AB.

From this we get CD – AB = BD – AC ----------(2)

From (1) and (2) we get AB – CD = CD – AB which will be 0.

This results in AB = CD and from (1) AC = BD. By taking the pairs of triangles BCD, CDA and ABC, DAB we get the other pair of opposite sides BC and AD to be equal.

We have two conditions: Opposite sides are equal, and diagonals are equal.

Hence it is a rectangle.

Answer: C

24) Adding 1 to the product of four consecutive positive integers always results in a perfect square. The first 2022 such square numbers are:

$$1 \times 2 \times 3 \times 4 + 1 = 25 = 5^2$$
$$2 \times 3 \times 4 \times 5 + 1 = 121 = 11^2$$
$$3 \times 4 \times 5 \times 6 + 1 = 361 = 19^2$$
$$\cdots$$
$$2022 \times 2023 \times 2024 \times 2025 + 1 = 16765347891601 = 4094551^2$$

In the list of 2022 numbers 5, 11, . . ., 4094551 whose squares are found in this way, how many have last digit equal to 1? _______.

Solution:

We have $n(n+1)(n+2)(n+3)+1=[n^2+3n][n^2+3n+2]+1$

$$=[(n^2+3n+1)-1]\cdot[(n^2+3n+1)+1]-1$$

$$=(n^2+3n+1)^2-1+1=(n^2+3n+1)^2$$

Therefore, $(n+1)(n+2)(n+3)(n+4)+1=[(n+1)^2+3(n+1)+1]^2$

The two successive bases on RHS of the equations differ by $2n\ +\ 4$.

Since $(n+1)^2+3(n+1)+1-[n^2+3n+1]=2n+4$

Therefore, the sequence of bases will be 5, 11, 19, 29, 41, 55, 71, 89, ….

∴ The end digits of the bases of perfect squares form a cyclic pattern as follows:

5, 1, 9, 9, 1, 5, 1, 9, 9, 1….

Cycle repeats for every 5 terms.

Exactly two bases end with digit '1' for every 5 bases.

Therefore, there will be $\frac{2}{5}\times 2020$ = 808 bases that ends with digit '1' in the first 2020 numbers. 2021st base ends with digit '5' & 2022nd base ends with digit '1'.

Therefore, there will be 808+1=809 bases that ends with digit '1' in the first 2022 numbers.

Answer: 809

25) In $\triangle\,ABC, AB\ =\ 52$ *unit*, $AC\ =\ 25$ *unit* and $BC\ =\ 33$ *unit.*

AD is the altitude to side BC.

The difference in length between AD and CD is ______ (*unit*).

Solution:

This triangle is an obtuse angled triangle since $52^2>33^2+25^2$.

Note: $52^2-25^2=(52+25)(52-25)=77\times 27=7\times 11\times 3\times 9$

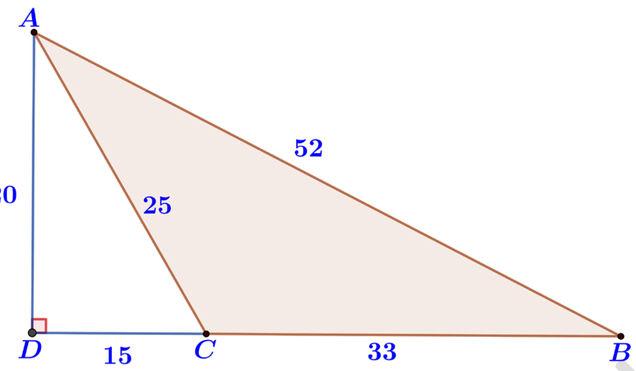
$$52^2-25^2\ =\ 33\ \times 63\ >\ 33\ \times 33.$$

The altitude AD falls outside BC as $\angle ABC$ is obtuse.

By heron's formula, we can find the area of triangle as

$$Area(\triangle\,ABC)=\sqrt{55\times(55-52)\times(55-25)\times(55-33)}$$
$$=\sqrt{5\times 11\times 3\times 3\ \times 10\times 2\ \times 11}=\sqrt{11^2\times 3^2\times 10^2}$$
$$=\ 330\ sq.\ unit.$$

$\therefore \frac{1}{2} \times BC \times AD = 330 \Rightarrow \frac{1}{2} \times 33 \times AD = 330 \Rightarrow AD = 20$ unit.



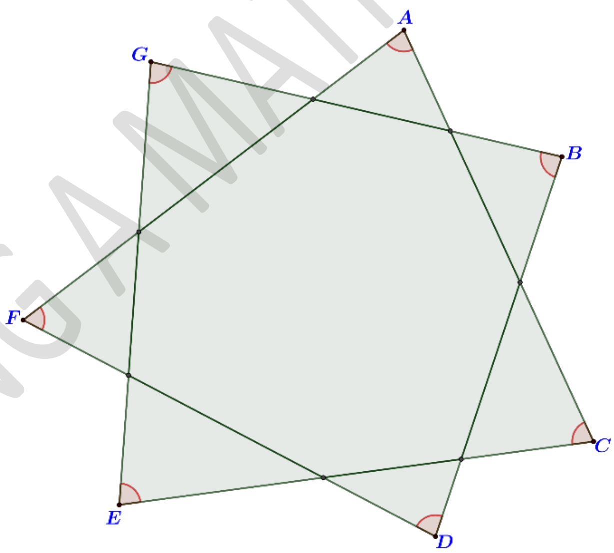
By Pythagoras theorem, we have $AC^2 - AD^2 = BD^2$.

Therefore, $BD^2 = 25^2 - 20^2 = 5^2(5^2 - 4^2) = 5^2 \times 3^2 = 15^2$.

So, BD = 15 unit. Hence AD – CD = 20 – 15 = 5 unit. **Answer: 5**

26) In the given star what is the measure of ∠A+∠B+∠C+∠D+∠E+∠F+∠G?

________(in degrees).



Solution: ∠A+∠B+∠C+∠D+∠E+∠F+∠G = sum of the vertex angles of the 7 triangles formed in the figure.

Sum of the angles of all the 7 triangles = 7×180°. = 1260°.

Sum of all the base angles of the 7 triangles = 2 × sum of the exterior angles of the inner heptagon = $2 \times 360° = 720°$.

Hence the required angle sum = 1260° – 720° = 540°. **Answer: 540**

27) The smallest positive integer that leaves remainders 3, 2, 1 when divided by 10, 9, 8 respectively is ------.

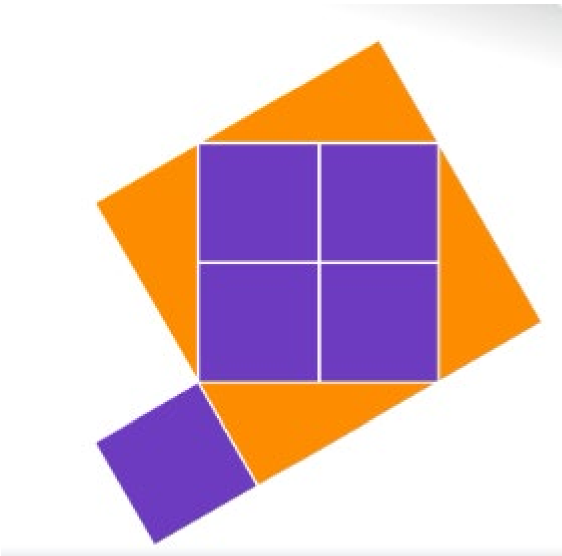
Solution: Any common multiple of 10, 9, 8 when subtracted with 7 satisfies the required condition. Note that 10 – 3 = 9 – 2 = 8 – 1.

To get the smallest such number take the LCM of 10, 9, 8 which is 360.

The required number is 360 – 7 = 353

Answer: 353

28) If the length of the purple squares is 1 unit, what is the square of the length of the longer perpendicular side in the yellow right triangles? ____ $unit^2$.



Solution: The smaller side is 1 unit. The hypotenuse is 2 units.

Hence the square on the longer leg of the yellow triangle is $2^2 - 1^2 = 3$ sq,units.

Answer: 3

29) How many perfect squares < 2022 are there such that each digit in it is the square of a whole number?

A) 11 B) 8 C) 9 D) 10

Solution: All single digit perfect squares, 1, 4, 9 must be counted. In 2-digit perfect squares, only 49 satisfies the condition. Let us look at 3-digit perfect squares:100, 144, 400, 441, 900. The only 4-digit perfect square < 2022 is 1444. Hence, we have 10 such perfect squares. **Answer: D**

30) Arjun says: “Balaji is lying.”

Balaji says: “Chandra is lying.”

Chandra says: “Dravid is lying.”

Dravid says: “Arjun is lying.”

How many of them are lying?

A) 1 B) 2 C) 3 D) 4

Solution: Note that the statements are cyclical. Assume Arjun tells the truth. So, Balaji is lying. So, Balaji’s statement is false. So, Chandra tells the truth and hence Dravid is lying. Thus, Arjun tells the truth as per our assumption.

Hence, two people are lying.

As the statements are cyclical wherever you start you will get two people lying and two telling the truth.

Answer: B