THE DEEPER YOU GO, THE MORE YOU GROW!

RAISING A MATHEMATICIAN FOUNDATION

Solutions for SQUARE ROOTS

1. 30. Ways to distribute 4 laddus between Ram and Lakhan is 5 (0, 1, 2, 3, 4 to Ram and rest to Lakhan) and ways to distribute 5 kaju kathlis between Ram and Lakhan is 6 (0, 1, 2, 3, 4, 5 to Ram and rest to Lakhan). Total number of ways is 5 x 6 = 30

   Answer: 30

2. **Solution:** As we want least number of squares to cover the rectangle, let us first fit 24 x 24 Squares. Three can be fitted covering 72 x 24 rectangle. Left over piece is 10 x 24. We can cover it with two 10 x 10 squares. Left over piece now is 4 x 10. This can be covered with two 4 x 4 pieces and two 2 x 2 pieces. Totally we used 9 pieces. This is the least.

   Answer: 9

3. The sum of all prime numbers between 100 and 130 is 660. The difference between the largest and the smallest among them, is --------.

   Solution: Smallest prime between 100 and 130 is 101. Largest prime is 127. The difference is 26. (There are 6 primes between 100 and 130; They are 101, 103, 107, 109, 113, 127)

   Answer: 26

4. **Solution:** Sum of numbers from 1 to 26 = 351. Take 15 and 21 two multiples of 3. Their product is 315. 351 – 15 – 21 = 351 – 36 = 315. Product of the two numbers is 315.

   Answer: 315

5. **Solution:**

The first ladder has 1 step and 5 sticks. Second one has 2 steps and 3 sticks. Observing a pattern, we see:

1st figure has 1 step ladder made of 5 sticks.
2nd figure has 2 steps ladder made of 8 sticks.
3rd figure has 3 steps ladder made of 11 sticks.
4th figure has 4 steps ladder made of 14 sticks.
5th figure has 5 steps ladder made of 17 sticks.

⋮

a th figure has a steps ladder made of $(3a + 2)$ sticks.

We need to equate $3a + 2 = 101$ and we get $a = 33$.

One can use the formula of sum of arithmetic progressions and can still solve it.

Answer: 33

6. The least value of x + y is 8. Given that x > 4 and 3 divides 7x + 4, x = 5 is the least value satisfying this condition. If x = 5 then 5 divides 3y + 1 if y = 3. Hence x + y = 8.

   Answer: 8

7. 16 sq. unit. If PB = SC = a unit, then, 40/(8 – a) = 54/(10 – a). Solving we get a = 16/7 unit and AD = QR = 7 unit. Hence the required area is 16 sq. unit.

   Answer: 16

8. 81.

   Let the number of parked cars, number of parked two wheelers, number of parked bicycles be c, t and b respectively.

   Given that $c + t = 156$; $t + b = 175$ and $c + b = 143$.
   Hence, $2(c + t + b) = 474$. Therefore, $c + t + b = 237$.

   Number of bicycles parked = $b = 237 - 156 = 81$.

   Answer: 81

9. 12. As one step of each length is a must, 14 – 2 – 3 – 4 = 5 is the steps that are to be fixed now. Clearly it can be done only with one 2 and one 3. So, we have 2 S, 2 M and 1 L steps to be arranged, satisfying the required conditions. With no conditions number of steps is 30. If both S's come together there will be 12 cases and for two M's also the same number. But these 24 will cover both S's and M's coming together namely 6. Thus, we have 30 – 12 – 12 + 6 = 12 cases.

   Answer: 12

10. **Solution:** In N rows we get a total of N(N+1)/2 as all rows are full. N(N+1)/2 = 6216. As N(N+1) = 12432 > 10000, N must be > 100. Using the units digit of 12432 we get N = 111, since 111 × 112 = 12432. Sum of the digits of N is 3.

    Answer: 3

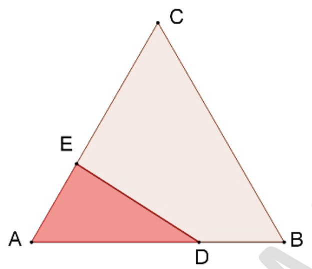
11. **Solution:** LCM(a, b) = $2^5 \times 3$; LCM(b, c) = $2^5 \times 5^2$; LCM(c, a) = $2^4 \times 3^1 \times 5^2$. 5 cannot be a prime factor of b, and 3 cannot be a prime factor of b from what is given. Hence b must be a power of 2 only. 5 is not a prime factor of a, as it is not found in LCM(a, b). Therefore, 3 is a factor of a. Hence, a = $2^k \times 3$ and b = 2^5, c = $2^m \times 5^2$. Power k takes values 0, 1, 2, 3, 4 only as LCM(c, a) = $2^4 \times 3^1 \times 5^2$. If m = 4, then for these values of k, LCM(c, a) = $2^4 \times 3^1 \times 5^2$. If k = 4, then m can be any number from 0 to 3. So totally we have 9 triples (a, b, c) only.

Answer: 9

12. **Solution:** ABCD (S_1) is a square inscribed in a circle of diameter 10 cm. Area of ABCD = 100/2 = 50 cm^2. EFGH (S_2) is the square inscribed in the semicircle. OG = 5 cm. OF = 2.5 cm. Hence GF^2 = area of EFGH = $5^2 - 2.5^2 = 25 - 6.25 = 18.75$ cm^2. Hence, M = $\frac{Area\ of\ S_1}{Area\ of\ S_2} = \frac{50}{18.75} = \frac{200}{75}$. Hence 75 × M = 200

Answer: 200

13.



Solution: Let h be the altitude from C to AB. Then The altitude from E to AD in ΔADE is h/3. Area of ΔADE = ½ ×AD×h/3 = ½ × $\left(\frac{2}{3}\right)$AB×h/3 = (2/9) × area of ΔABC. $\frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta ABC} = \frac{2}{9}$.

Answer: D

14. **Solution:** $5^{\frac{1}{4}} . 5^{\frac{3}{4}} . 5^{\frac{5}{4}} \ldots . 5^{\frac{2n+1}{4}} = 5^{\frac{(n+1)^2}{4}}$ >100,000 and $5^7 < 100{,}000 < 5^8$. Hence $\frac{(n+1)^2}{4} \geq 8$, i.e., $(n+1)^2 \geq 32$. Least value of n is 5 as $5^2 < (n+1)^2 < 6^2$

Answer: 5

15. What is the missing digit A if the 17-digit number A0413263952657891 is exactly divisible by 13? ---------.

Solution: A0413263952657891 = A04×10^{14} + 13×10^{12} + 26×10^{10} + 39×10^8 + 52×10^6 + 65×10^4 + 78×10^2 + 91. 13, 26, 39, 52, 65, 78, 91 are all multiples of 13. Therefore A04×10^{14} and hence A04 must be a multiple of 13 since HCF(13, 10) = 1. Thus A = 1. **Answer: 1**

16. If the perimeter of a rectangle is $44\sqrt{2}$ cm, then the least possible length of the diagonal in cm is ------

Solution: a and b are the sides of the rectangle, then $a + b = 22\sqrt{2}$. Let the sides be $11\sqrt{2} - x,\ 11\sqrt{2} + x$. Therefore, $(\text{Diagonal})^2 = 2(242 + x^2)$. This is least if $x = 0$. Least diagonal value is $\sqrt{484} = 22$.

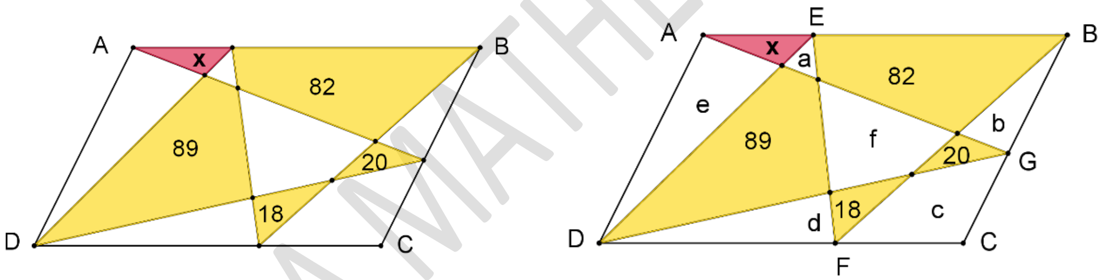
Answer: 22

17. The value of $\sqrt[4]{\dfrac{16^8+4^{10}}{16^6+8^4}}$ is ------

Solution: The numerator $16^8 + 4^{10} = 4^{16} + 4^{10} = 4^{10}(4^6 + 1)$. The denominator is $16^6 + 8^4 = 4^{12} + 4^6 = 4^6(4^6 + 1)$. Hence the given expression reduces to $\sqrt[4]{4^4} = 4$.

Answer: 4

18. ABCD is a parallelogram. If the areas of the yellow regions are as marked in the figure, then 10 times the area of the region marked x is -------.



Solution: It is given that ABCD is a parallelogram. Hence,
area(AGD) = area(AGB) + area(DGC). Therefore,
$e + 89 + f + 20 = x + a + 82 + b + d + 18 + c$:
$e + f + 9 = a + b + c + d + x$ ------ (1)
area(ADE) + area(EFB) = area(DEF) + area(FBC). Therefore,
$e + x + 18 + f + 82 = d + 89 + a + c + 20 + b$:
$e + x + f = a + b + c + d + 9 = e + f + 9 - x + 9$ from (1)
Hence $2x = 18$ and $x = 9$. Hence $10x = 90$.

Answer: 90

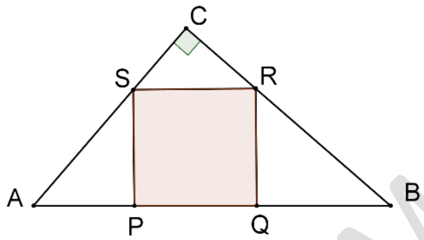
19. It is given that $(x - 2)^2 + (y + 6)^2 = 169$ where x and y are *integers*. Then the minimum value of $x^2 + y^2$ is -------.

Solution: $(x - 2)^2 + (y + 6)^2 = 169$. $169 = 13^2 = 12^2 + 5^2 = 13^2 + 0^2$ are the only two ways we can write 169 as the sum of squares of integers. Possibilities for x, and y are listed below:

(x – 2)	(y + 6)	x	y	$x^2 + y^2$
13	0	15	–6	261
–13	0	–11	–6	157
0	13	2	7	53
0	–13	2	–19	365
12	5	14	–1	197
12	–5	14	–11	317
–12	5	–10	–1	101
–12	–5	–10	–11	221
5	12	7	6	85
5	–12	7	–18	383
–5	12	–3	6	45

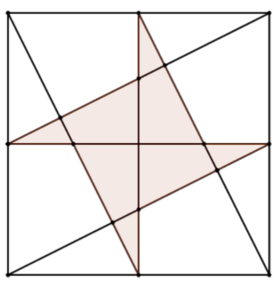
Least value is 45. **Answer: 45**

20. ABC is a right-angled triangle where $\angle ACB = 90^o$. P, Q, R, S are points on sides AB, AB, BC, CA respectively such that PQRS is a square. If CS = 36 units and QR = 60 units, then the perimeter of ΔABC is ------ units.



Solution: RS = 60 units since PQRS is a square. $CR^2 = 60^2 – 36^2 = 96 \text{ x } 24 = 48^2$. CR = 48. The altitude from C to SR = 36×48/60 = 28.8 units. Hence from similar triangles CSR and CAB, we get $\frac{CS}{CA} = \frac{28.8}{88.8}$. CA = 888 ×36/288 = 111. Perimeter of ΔCSR = 36 + 48 + 60 = 144 units. Perimeter of ΔABC = 144× 111/36 = 444 units. **Answer: 444**

21. In the diagram, a corner of the shaded star is at the midpoint of each side of the large square. If the side of the large square is 10 cm, then the area covered by the shaded star in cm^2 is -------



Solution: The bigger square is divided into 4 equal smaller squares of side 5 cm. The shaded region is made up of 4 equal non-overlapping right triangles, each clearly one fourth the area of the smaller square. Totally we get the area of the shaded region = area of the smaller square = 25 cm^2.

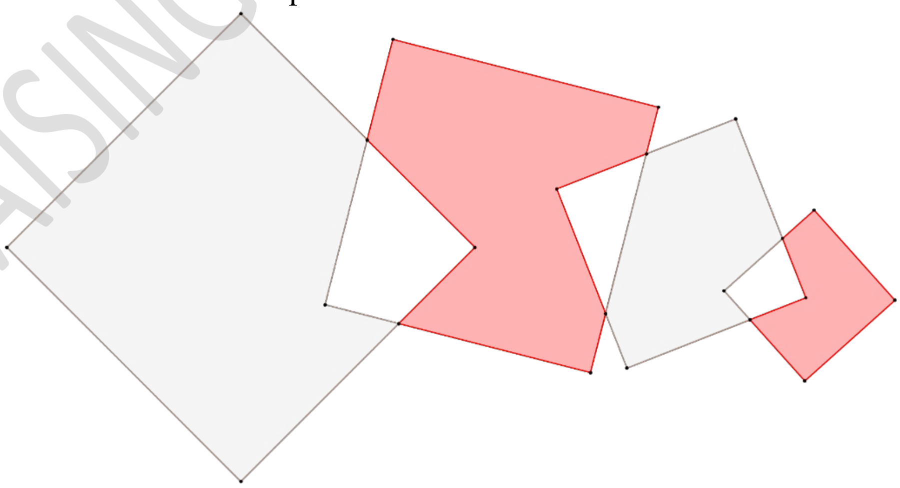
Answer: 25

22. The sum of the interior angles of a convex polygon excluding one angle is 2021°. Then the excluded angle measure in degrees is -------.

Solution: 139
The sum of the angles of an n-sided polygon is (n – 2)180° > 2021. All angles are less than 180° in a convex polygon. Hence n > 13. The least such n is 14.ABC = 90 +x
Hence the excluded angle is 180 × 12 – 2021= 139°. If a larger n is taken we will get a reflex angle, not acceptable. **Answer: 139**

23. **Solution:**
Consider a smaller even number grid, say 6 x 6 grid. You will see that the numbers on the top left corner and bottom right corners will be the first and the last. In this case, their sum is 36+1=37. If you look at the bottom right corner and top right corner, the numbers' sum will also be the same (37 in this case). This will happen if you take a particular row number from the bottom (say, 2nd row from the bottom and second row from the top) and a particular cell from the right (say, 3rd cell from the right in both rows). Their totals will be 37 in a 6x6 grid. So, if you take a 100 x 100 grid, the totals of the four centermost cells will be 10001+10001=20002 **Answer: 20002**

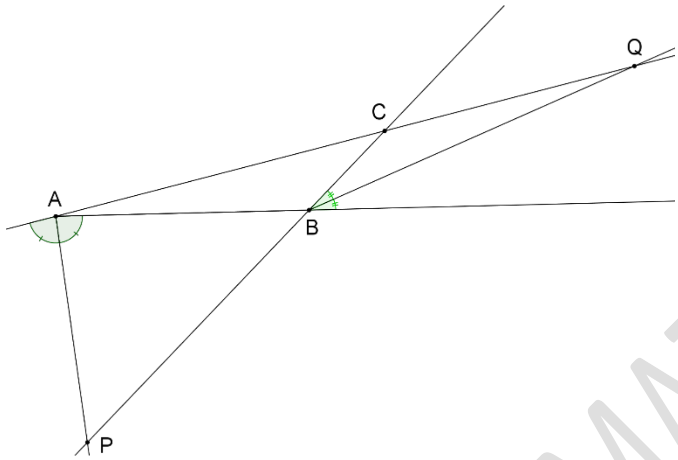
24. The figure shows 4 overlapping squares which have sides of lengths: 6 unit, 7 unit, 8 unit and 9 unit. What is the difference between the total area shaded grey and the total area shaded red in sq. unit?



Solution: 30 sq. units.
Let the area of the first white region be x, second be y and third be z.
Sum of the areas of the 1st and 3rd squares is 81 + 49 = 130 = grey area + x+y+z.

Sum of the areas of the 2nd and 4th squares is 64 + 36 = 100 = red area + x+y+z. Difference is 130 – 100 = 30 sq. units. **Answer: 30**

25. In ΔABC, the angle bisectors of the exterior angles ∠A and ∠B intersect their opposite sides at P and Q respectively. If AP = AB = BQ then ∠BAC = -------.



Solution: ∠BAC = 12°

Let ∠BAC = x. From ΔABQ, as AB = BQ, ∠BQA = x.

∠BAP = 90° – x/2 as AP is the external angle bisector of ∠A.

Hence, ∠APB = ∠ABP = ½ (180° – (90° – x/2)) = 45° + x/4.

∠ABC = 180° – ∠ABP = 135° – x/4.

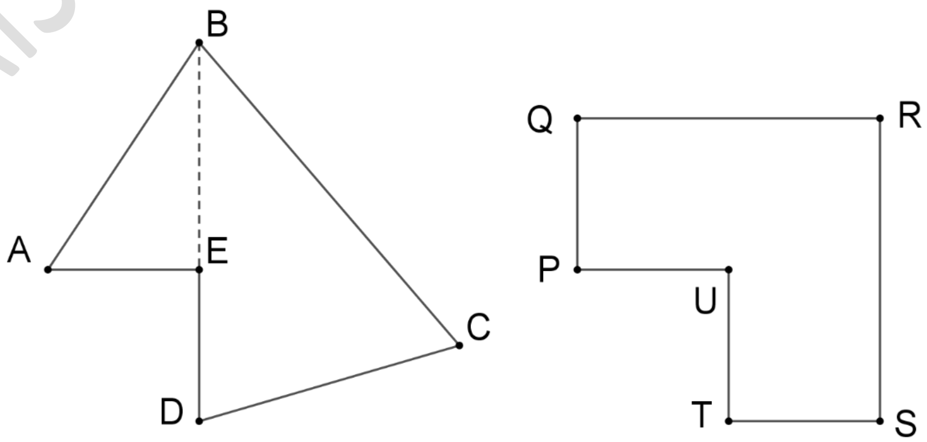
∠ABQ = ∠ABC + ∠CBQ = 135° – x/4 + ½∠ABP = 135° – x/4 + ½ (45° + x/4)

= 157.5° – x/8 = 180° – 2x. (From ΔABQ) Hence, 15x/8 = 22.5; x = 12°.

Answer: 12

26. Which of the following is false?

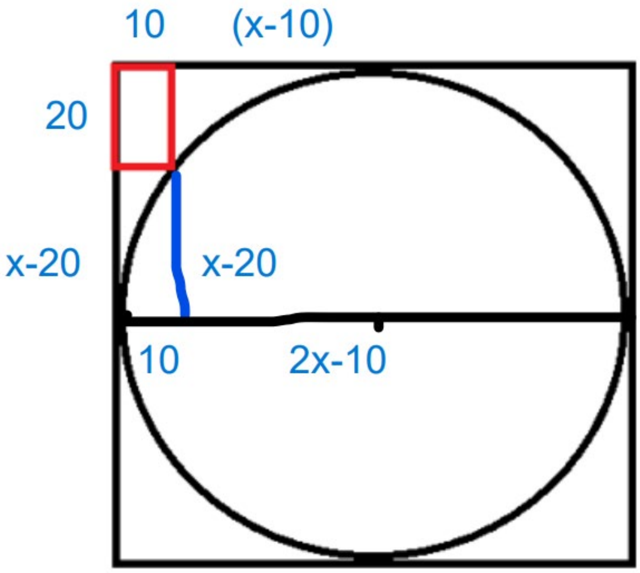
A. There exists a pentagon where one of its sides is a part of one of its diagonals.
B. There exists a hexagon where every pair of adjacent sides are perpendicular.
C. There exists a quadrilateral where its four sides and two diagonals are all equal in length.
D. None of these. **Solution: C**



From these we see that A and B are true. Hence the answer is C. **Answer: C**

27. **Solution:**

There are different ways to solve this problem. One is by looking at the below construction and figuring out that x-20 will be the geometric mean of 10 & 2x-10. Thus, we get (x-20)^2 = x(2x-10). But if you don't know this fact about Geometric Mean, you can solve it using the Pythagorean Theorem.

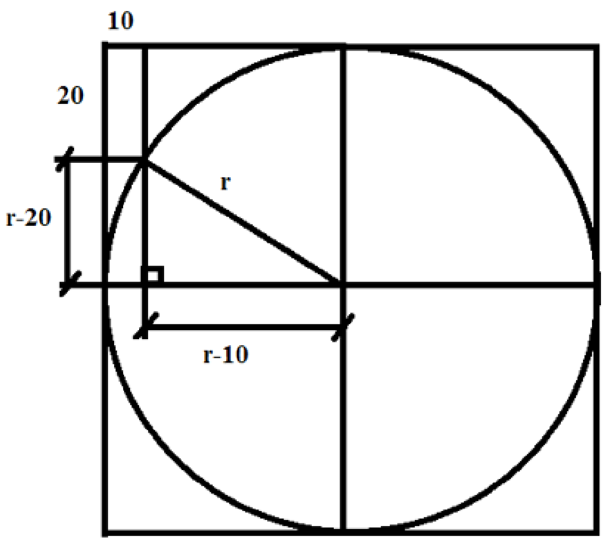


Solution using the Pythagorean Theorem:

In the figure below, applying the Pythagorean Theorem we get, $r^2 = (r-20)^2 + (r-10)^2$

By solving we get 2 values of r = 10/50

So, value of radius 10 is not possible so value of radius must be 50.



Answer: 50

28. In a right triangle the length of the hypotenuse is 5 units and the sum of the lengths of the other two sides = $\sqrt{29}$. Then the area of the triangle is ------.

Solution: Let a, b the lengths of the legs and c the hypotenuse of the triangle. Then c = 5, and $c^2 = 25 = a^2 + b^2$. It is given that $a + b = \sqrt{29}$, $(a + b)^2 = 29 = a^2 + b^2 + 2ab = 25 + 2ab$. Then 2ab = 4 and hence area of the triangle (1/2)ab = 1 sq. unit. Area is 1 sq. unit.

Answer: 1

29. **Solution:**

Let us name the coordinates as (x,y). Thus, in (1,3) x= 1 and y=3. If you take (1,3) and any point **on a lower line** (like (3,2), (5,1) AND are to the right direction of the initial point (1,3), then the shortest distance between them is the difference between the x co-ordinate of both the points. Since (51,10) & (15,21) are of the same kind, the minimum distance between them will be 51-15=**36**.

Answer: 36

30. A daily labourer is paid Rs 200 for each day he works.

He is fined Rs 50 if absent on a working day.
Every Thursday is a holiday for the labourer.
No pay and no fine on holidays.
In a particular month, a labourer worked either a full day or absent the whole day. The labourer gets Rs 4450 in total for that month.
The number of days he worked during that month, was ------.

Solution: He worked for 23 days only. As he was absent for 3 days, he was fined Rs 150. Salary he got was 23 x 200 – 150 = 4600 – 150 = Rs 4450

Answer: 23