THE DEEPER YOU GO, THE MORE YOUR GROW

RAISING A MATHEMATICIAN FOUNDATION

Solutions for CUBE ROOTS

1. Anu, Bama, Chitra and Deepa ate 50 Gulab jamuns. Each ate at least two. Deepa ate more than each of the other three. Anu and Bama ate 31 Gulab jamuns. How many did Chitra eat? ______.

   Solution: As total number of Gulab jamuns is 50 and Anu and Bama ate 31 of them, Chitra and Deepa ate 19 jamuns altogether. Deepa ate more than all the three. This means Deepa could have eaten a maximum of 17 jamuns (as each one of them eats at least two). Is this valid? Yes. This means Anu and Bama might have eaten 15 and 16 jamuns between them. So, Deepa must have consumed 17 jamuns implying Chitra ate just two jamuns.

   Answer: 2

2. Which among the following products is the greatest?

   A. 333333×444444 B. 222222×666667 C. 333334×444443

   D. 222223×666666

   Solution:

   Let $y = 333333 \times 444444 = 111111 \times 3 \times 111111 \times 4 = 666666 \times 222222.$

   $$\begin{aligned}\text{Then } 222222 \times 666667 &= 222222 \times (666666 + 1)\\ &= 222222 \times 666666 + 222222 = y + 222222\end{aligned}$$

   $$\begin{aligned}333334 \times 444443 &= (333333 + 1) \times (444444 - 1)\\ &= y + 444444 - 333333 - 1\\ &= y + 111110\end{aligned}$$

   $$\begin{aligned}222223 \times 666666 &= (222222 + 1) \times 666666\\ &= y + 666666\end{aligned}$$

   The greatest product is 222223 x 666666.

   Answer: D

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3. The product of 2^{2022} and 5^{2022} is written in the decimal numeral, then the total number of digits in that product is _______.

Solution:

Let $1\underbrace{0000\ldots00}_{m\ zeros} < 2^{2022} < 1\underbrace{0000\ldots00}_{(m+1)\ zeros}$ and $1\underbrace{0000\ldots00}_{n\ zeros} < 5^{2022} < 1\underbrace{0000\ldots00}_{(n+1)\ zeros}$.

So, 2^{2022} lie between the smallest $(m+1)$ −digit number and the smallest $(m+2)$ −digit number. Therefore, 2^{2022} must be a $(m+1)$ −digit number.

5^{2022} lie between the smallest $(n+1)$-digit number and the smallest $(n+2)$ −digit number. Therefore, 5^{2022} must be a $(n+1)$ −digit number.

Now,

$$1\underbrace{0000\ldots00}_{m\ zeros} \times 1\underbrace{0000\ldots00}_{n\ zeros} < 2^{2022} \times 5^{2022} < 1\underbrace{0000\ldots00}_{(m+1)\ zeros} \times 1\underbrace{0000\ldots00}_{(n+1)\ zeros}$$

$$1\underbrace{0000\ldots00}_{(m+n) zeros} < 2^{2022} \times 5^{2022} < 1\ \underbrace{0000\ldots00}_{(m+n+2)\ zeros}$$

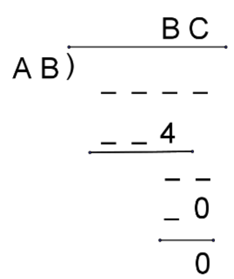
$$1\underbrace{0000\ldots00}_{(m+n) zeros} < 1\underbrace{0000\ldots00}_{2022\ zeros} < 1\underbrace{000000\ldots00}_{(m+n+2)\ zeros}$$

We can observe that 10^{2022} is a digit 1 that should be followed by $(m+n+1)$ zeros. Therefore, $m + n + 1 = 2022$.

The total number of digits written in both the numbers 2^{2022} and 5^{2022} is (m+1) +(n+1) = 2023.

Answer: 2023

4. Look at the following division where A, B, C are different digits with C≠ 0. A blank space represents any digit. The Sum : A + B + C is ______.



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Solution: Note that the product of two 2-digit numbers AB and BC is a 4-digit number. Moreover, AB x B has 4 as its unit's place.

This implies B is either 2 or 8.

Case (i): B = 8.

Then C must be 5 for the product AB × C to have unit's digit 0.

Further if A > 1, then the product AB × C will have three digits which is not the case. Hence A = 1 and C = 5. The dividend is 18 × 85 = 1530.

A + B + C = 14.

Case (ii): B = 2.

C must be 5 for the same reason.

Only for A = 1 we will get 12 × 5 = 60 a 2-digit product.

But then 12 × 2 = 24, a 2-digit number a contradiction.

So, no solution exists with B = 2.

Answer: 14

5. I am a 2-digit number. My ten's digit is greater than the one's digit.

   The difference between the digits is greater than 3.

   The sum of the digits is greater than 11. The number is a multiple of 14.

   Who am I? _____.

   Solution: Let AB be the 2-digit number.

   Given conditions translate to A – B > 3 and A + B > 11. This implies A > 7.

   Look for multiples of 14 in eighties and nineties, according to the given third condition. It is 84 and it satisfies all the conditions.

   Answer: 84

6. The sum of the dates of the Thursdays in a month is 80.

   What is the maximum number of months that this can happen in a leap year? ___.

   Solution: Note that all the dates 1, 8, 15, 22, 29 have the same day.

   The sum of these dates is 75. Let us look at the dates 2, 9, 16. 23, 30 which also will all fall on the same day and their sum is 80.

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So, these are the dates we must look at.

Hence 2 must fall on a Thursday meaning the first of the month is a Wednesday. Let us take January first as Wednesday. Then, being a leap year, first of April will be 31 + 29 + 31 = 91 days away and will be a Wednesday. Again, July first will be 30 + 31 + 30 = 91 days away and will be a Wednesday. After that no other months first falls on a Wednesday.

So, we have three months, January, April, and July where the dates of the Thursdays add up to 80!

Answer: 3

7. In the subtraction given the top four boxes are filled by 3, 5, 6, 7 in some order and the circles in the second row are filled with 3, 6, 8, 9 in some order.

   The answer is 3487.

   Then, what is the number represented by the boxes in the top row? _______.

   □ □ □ □
   − ○ ○ ○ ○
   ―――――
   3 4 8 7

Solution: The unit's places in the two rows will be 6 and 9 resulting in 7 on subtraction after borrowing from the ten's place. In the tens place we need 8 in the answer and hence in the top row 5 will be there and in the second row 6 will be there in ten's position.

For the ten's position also, we need to borrow from the hundred's position. Hence in the hundred's position we have 3 and 8 to get a 4 in the answer after borrowing one from thousands. Finally, we place 7 and 3 in the thousands place. The top number is 7356.

Answer: 7356

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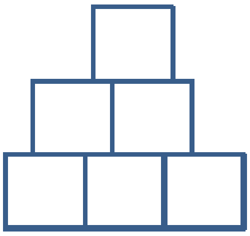
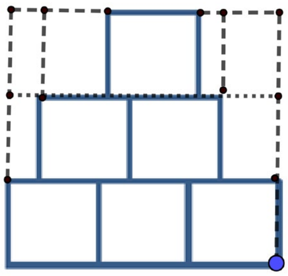
8) The figure made of six identical unit squares (square with side length of 1 cm), as shown.



What is the perimeter of this figure? ____ cm.

Note: Perimeter of a closed figure is the total length of its boundary.

Solution: The perimeter of the given figure is the same as that of the bigger square enclosing it. Observe and Imagine!



Therefore, the perimeter of the figure = $4 \times 3\ cm = 12\ cm$.

Answer: 12

9) Place a single digit in each empty square in the diagram so that each row, each column and each diagonal contain all numbers from 1,2,3,4 and 5. Then, the value of # is ___.

	2			
				#
		4		
			1	
5				

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Solution: Consider the cells marked with A, B, C as shown.

	2			*C*
			B	#
		4		
	A		1	
5				

A cannot be 4 or 5 as the diagonal is occupied with these numbers.

A cannot be 2 as it is occupied by its column.

A cannot be 1 as it is occupied with its row. So, A can take only 3.

Now B cannot be 3, 4 or 5 as the diagonal is occupied with these numbers.

B cannot be 1 as it is occupied by its column. So, B can take only 2.

Therefore, C must take the value 1. Now, the diagram will look like ↙

D	2			1
	E		2	#
		4		
	3		1	
5				*F*

Consider the cells marked with D, E, F as shown. D cannot be 1,2, 4 or 5.

D must be 3. Now E cannot be 1,2,3 or 4. E must be 5.

Therefore, F must be 2. Now, the diagram will look like ↙

3	2	*G*	*H*	1
	5		2	#
		4	*J*	
	3		1	
5			*I*	2

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Consider the cells marked with G, H, I as shown.

Now G cannot be 1,2 3 or 4 and it must be 5 only. $\therefore$ H = 3.

I cannot be 1,2,3 or 5 and it must be 4 only. $\therefore$ J = 5.

Now, the diagram will look like ↓

3	2	5	4	1
	5		2	#
		4	5	*L*
	3		1	*K*
5			3	2

Consider the cells marked with K, L as shown.

Now L, # cannot be 5 and therefore K must be 5.

L cannot be 4. Therefore, L = 3. Hence, the value of # must be 4 only.

Now, the diagram will look like ↓

3	2	5	4	1
	5		2	# 4
		4	5	3
	3		1	5
5			3	2

Answer: 4

You may continue filling the remaining cells to verify whether the condition is satisfied ✓

3	2	5	4	1
1	5	3	2	# 4
2	1	4	5	3
4	3	2	1	5
5	4	1	3	2

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Alternate Solution:

Look at the diagonal from the bottom left to top right corners. In the 4th row second column cell you can have only 3 as 1 and 2 are not possible from the given data. In the second row, 4th column, 1 cannot occur. It must be 2. So, the top right cell is 1. This completes one diagonal. We get the figure below:

	2			1
			2	#
		4		
	3		1	
5				

Stage 1

3	2			1
	5		2	#
		4		
	3		1	
5				2

Stage 2

3	2			1
	5		2	#
	1	4		
	3		1	
5	4			2

Stage 3

3	2			1
1	5		2	#
2	1	4		
4	3		1	
5	4			2

Stage 4

Now, look at the diagonal from top left to bottom right. 2 cells are occupied. 2 cannot occur in the top left cell and the one in the 2nd column, 2nd row cell. So, 2 is placed in the bottom right cell. Hence 5 is placed in the 2nd column, 2nd row cell. So, 3 occupies the top left cell.

In the second column, 4 occurs in the last row. Hence, the 3rd cell in second column is 1. This gives column 1 2nd cell as 1, 4th cell as 4 and 3rd cell as 2.

Now the first row gets filled with 4 in the fourth cell and 5 in the 3rd cell (red).

The fourth-row 5th cell will be 5 and hence 3rd cell will be 2 (blue).

Third column second cell will be 3 (green). Hence # will be 4.

Final solution is given below:

3	2	5	4	1
1	5	3	2	# 4
2	1	4	5	3
4	3	2	1	5
5	4	1	3	2

Answer: 4

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10) Between 180 and 220, there exist eleven consecutive natural numbers that are all composite numbers. What is their sum? _________.

Solution:

Any even number in this range is composite.

We need to look for 5 consecutive odd numbers that are all composite.

205 is a multiple of 5. 201, 207 are multiples of 3, $203 = 210 - 7$ is a multiple of 7 and $209 = 220 - 11$ is a multiple of 11.

Therefore, from 200 to 210, all are composite numbers.

Hence the sum of these 11 composite numbers from 200 to 210 is 2255.

Note: We can trust the information and no need to check for another possible combination of eleven consecutive composites as there are primes such as 181, 191, 193, 197, 199, 211.

Answer: 2255

11) All possible fractions are formed, with both numerator and denominator being single digit numbers.

$$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$$

$$\frac{2}{1}, \frac{2}{2}, \frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{2}{6}, \frac{2}{7}, \frac{2}{8}, \frac{2}{9}$$

.

.

.

$$\frac{9}{1}, \frac{9}{2}, \frac{9}{3}, \frac{9}{4}, \frac{9}{5}, \frac{9}{6}, \frac{9}{7}, \frac{9}{8}, \frac{9}{9}$$

How many fractions among them each having a value less than $\frac{1}{2}$? ____.

Solution: For denominators 1 and 2 no such fractions exist.

For denominators 3 and 4 it is one each ($<\frac{1}{2}$), for 5 and 6 as denominator there are two fractions each, denominator 7 and 8, three fractions each and for denominator 9 four fractions. Totally 1 + 1 + 2 + 2 + 3 + 3 + 4 = 16 fractions.

Answer: 16

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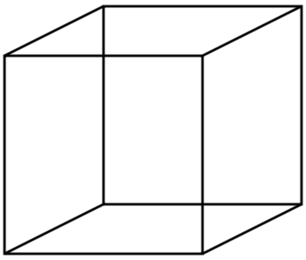
12) A cube has eight vertices (corners) and twelve edges, as shown.



Suppose all the vertices are sliced off such that all the initial vertices are removed and no edge is cut off completely. Let m be the number of vertices and n the number of edges in the new shape. What is the value of $(n - m)$? ___.

Solution: Each vertex is replaced by three new vertices.

So total vertices is m = 24. Original edges 12 in number stay. Moreover 24 new edges (three at each vertex) are created.

Total number of edges n = 12 + 24 = 36. Hence (n – m) = 12.

Answer: 12

13) Pradeep says: "My age is an odd number of years." – Statement 1

Vishal says: "If I multiply my age with it, I get an even number of years." – Statement 2

Pradeep says: "The difference between our ages is odd number of years." – Statement 3

Vishal says: "My age is an odd number of years." – Statement 4

Pradeep says: "The HCF of our ages is 1 year." – Statement 5

In this conversation between *Pradeep* and *Vishal*, exactly one of the five statements is true. Which statement is true?

A) 4 B) 3 C) 1 D) 2

Solution: Given that exactly one of the five statements is true. Look at the table

Pradeep's age	Vishal's age	S1	S2	S3	S4	S5	Condition
odd	odd	True	False	False	True	Mb	Not satisfied
odd	even	True	True	True	False	Mb	Not satisfied
even	odd	False	True	True	True	Mb	Not satisfied
even	even	False	True	False	False	False	Satisfied

Note: Mb stand for "May be True or False".

We can realise that only statement 2 (S2) can be true with all other statements false. Therefore, the correct option is D) 2. **Answer: D) 2**

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14) Sharmila's car number is a 4-digit number ABCD.

Each digit A, B, C, D is a different prime number. The two-digit numbers (as a part of the car number) AB, BC, CD are composite numbers.

The car number when divided by 8, leaves remainder 1.

What is Sharmila's car number? _________.

Solution: A, B, C, D are 2, 3, 5, 7 in some order. 23, 53, 73 are primes.

Hence, they must not occur as AB, BC, CD. So, B, C or D cannot be 3.

Therefore, A must be 3. The possible car numbers are 3257, 3527 and 3572.

Of these only 3257 leaves a remainder 1, when divided by 8.

Therefore, Sharmila's car number is 3257.

Answer: 3257

15) A *sequence of natural numbers* whose *second digit* (from left to right) is 0, are written in ascending order as follows:

10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 101, 102, …., 2020, 2021, 2022.

Note that the *first term* of the sequence is 10. The sequence ends with the *last term* 2022. The *seventh term* is 70. The *eleventh term* is 101.

How many terms are there in the sequence? ________.

Solution: Starting from 2000 up to 2022, there are 23 numbers. There are 9 2-digit numbers. Let us count the 3-digit numbers of this kind. They are 100 to 109, 200 to 209, 300 to 309, …, 900 to 909. They are 90 in number.

Consider the 4-digit numbers. They must be from 1000 to 1099 only. After that the hundreds digit is nonzero. They are 100 in number.

Totally we have 23 + 9 + 90 + 100 = 222. **Answer: 222**

Alternate Solution:

Consider the sequence of first 222 natural numbers:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, … 221, 222.

Now put or insert a digit 0 in the second position from left to right of every number. This will give you the sequence

10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 101, 102, …., 2020, 2021, 2022.

Therefore, there will be 222 numbers in the sequence.

Answer: 222

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16) Let a be the smallest 4-digit multiple of 11, with different digits.

Let b be the largest 4-digit multiple of 11, with different digits.

What is the value of $(b-a)$? _______.

Solution: Smallest multiple of 11 with 4 *distinct* digits is $a = 1023$.

The largest 4-digit multiple of 11 is $b = 9867$.

Any bigger 4 digit multiple of 11 will have repeated digits.

Hence, $b - a = 8844$.

Answer: 8844

17) How many times in a day will the angle between the hour hand and the minute hand of a clock be 160 degrees? ____.

Solution: Every hour, there will be a position where the hour hand and the minute hand of the clock will make an angle of 160° between them.

Therefore, the number of times this happens in a day, is 24 as there are 24 hours in a day.

Answer: 24

18) *Four Prisoners* escape from a prison.

The *prisoners*, *Mr East, Mr West, Mr South, Mr North* head towards different directions after escaping.

o The escape routes were *North, South, East* and *West Roads.*

o None of the *prisoners* took the road which was their namesake.

o *Mr East* did not take the *South Road*.

o *Mr West* did not take the *South Road*.

o *The West Road* was not taken by *Mr East*.

What *Road* did *Mr West* take to make his escape?

A) East B) West C) North D) South

Solution: Let us use a table to solve this where the row headers are prisoners and column headers are roads. We put X in cells which are not possible routes for the prisoners. Hence the diagonal cells are marked X.

From the last three statements we mark three more.

Hence Mr. East takes North Road. Let us tabulate as follows ↙

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	East Road	West Road	North Road	South Road
Mr. East	X	X	✓	X
Mr. West	✓	X	X	X
Mr. North			X	
Mr. South			X	X

Therefore, all other entries in North column are X.

Only road left for Mr. West is East Road

Answer: A

19) A group of students is as follows:

One-third of the group was students under 12 years old (aged 11 years or less).

One-half of the group was students under 13 years old (aged 12 years or less).

Exactly 6 students were under 11 years old (aged 10 years old or less).

Exactly 12 students were above 12 years old (aged 13 years old or more).

The number of students in the group is _________.

Solution: Exactly one-half students are 13 years and above as one-half is 12 years and below. It is given that they are 12 in number.

Hence total number of students in the group is 24.

Answer: 24

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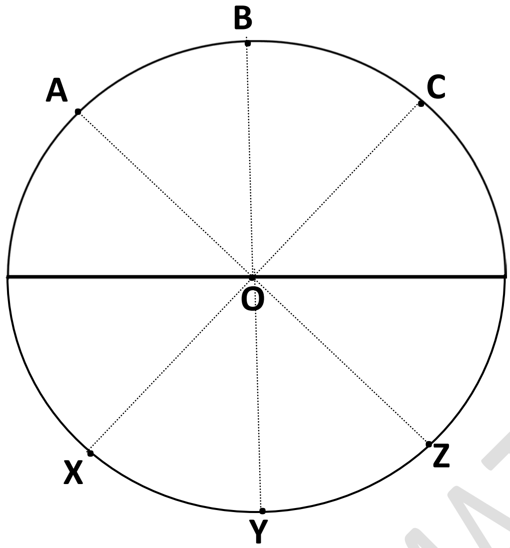
20) On a circle with centre O, points A, B, C, Q, Z, Y, X and P are such that they cut the circle into 8 equal parts. The measure of the angle XBZ is _____ (in degrees).



Solution: $\angle XOZ = 90^o$. $\angle XOB = 135^o$ and ΔXOB is isosceles with OX = OB. Hence $\angle XBO = \angle BXO = 22.5^o$. Similarly, on the other side $\angle ZBO = 22.5^o$. Therefore, $\angle XBZ = 45^o$.

Answer: 45 (in degrees)

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