Online Contest for 5^{th} and 6^{th} Standard

THE DEEPER YOU GO, THE MORE YOU GROW!

RAISING A MATHEMATICIAN FOUNDATION

Solutions for CUBE ROOTS

1. **Solution:**
Palani made a printing error of one digit in the report card of Aruna and the wrong total is 97 + 65 + 84 + 83 = 329. Actual total of Aruna was 323.
Actual total 323 is 6 less than the wrong total 329.
Therefore, the error must have happened in the unit's digit which is 6 or more.
The only possible units digit is 7 in the score of 97. The correct digit should be 1.
Therefore, the wrongly printed digit is 7.
Answer: 7

2. The pattern goes thus: 10, 16, 27, 33, 44, 50, …Note the successive differences in the odd position terms is 17 and the even position terms is also 17. Hence the 10^{th} term is $16 + 4 \times 17 = 16 + 68 = 84$.
Answer: 84

3. 38 terms. From the second term they are multiples of 9. 360, 351, …., 0. Up to 297 we subtract 9 which is the sum of the digits. From 297 the sum of the digits is 18 and we get 279 and then 261. From 261 we subtract the sum of the digits 9 successively to get 252, 243, 234, 225, 216, 207, 198. The sum of the digits now is 18. We get 180, 171, 162, 153, 144, 135, 126, 117, 108, 99, 90, 81, 72, …., 0. Hence number of terms is 38.
Answer: 38

4. Observe that $48 = 2 \times 2 \times 2 \times 2 \times 3 = \boxed{1} \times 1 \times \boxed{2} \times 2 \times \boxed{2 \times 2} \times 3$.
This can be rewritten as $48 = -1 \times 1 \times -2 \times 2 \times 4 \times 3$.
That gives us the product of 6 different integers to be 48 but the sum is 7 positive.
We need a negative sum.
So, we tune further to get $48 = -1 \times 1 \times -2 \times 2 \times -4 \times -3$.
Now the sum is negative. This is the only possible set of integers.
Hence the difference between the smallest and largest numbers among them is 2 – (-4) = 6.
Answer: 6

5. Given number is 2794, which is the sum of a number and its' digits. Clearly it must be a 4-digit number beginning with 2.

2699+2+6+9+9=2725 < 2794. Therefore, the number must be more than 2699.
Hundreds' placed digit must be 7.

2769+2+7+6+9=2793 < 2794. Therefore, the number must be more than 2769.
2780+2+7+8+0=2797 > 2794. Therefore, the number must be less than 2780.
It should be in 2770's and we have 277A+2+7+7+A=2794 giving A = 4.
Thus, we get the required number to be 2774. **Answer: 2774**

6. Since 37 and 13 are in the same column, in each row the number of elements is a factor of 37 – 13= 24.

E.gs are

1	2
3	4
5	6
⋮	⋮
13	14
⋮	⋮
37	38
⋮	⋮
49	50

1	2	3
4	5	6
7	8	9
⋮	⋮	⋮
13	14	15
⋮	⋮	⋮
37	38	39
⋮	⋮	⋮
49	50	

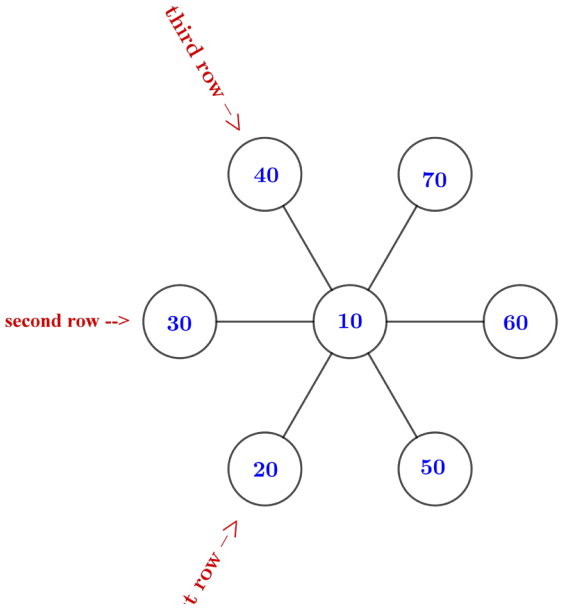
……

1	2	3	…	12	13	14	…	24
25	26	27	…	36	37	38	…	48
49	50							

2 is a factor of 24. 3 is a factor of 24. And so on …. 24 is a factor of 24.

The number of factors of 24 excluding 1 is 7. **Answer: 7**

7. 10. You can group them as (10, 20, 70); (10, 30, 60); (10, 40, 50).
10 being common to the three sets it must be placed in the centre.



Note: $\boxed{10}$+ 20 + 70 = $\boxed{10}$+30+60=$\boxed{10}$+40+50=100. **Answer: 10**

8. Given that Prof Kamakoti's car number is a 4-digit multiple of 4 made of digits 1,3,7 and another unknown digit.

The unknown digit must be even and should be the last digit of the car number.

It should be 2 or 6 only as 12,16,32,36,72 or 76 being the possible last two digits of the car number that is divisible by 4.

Given that the difference between the car number and its reverse is 4635.

Note: The car number might be bigger or its reverse bigger but their difference is 4635.

One can verify the possible combinations and can the find that only one combination works! That is, 6371 – 1736 = 4635. ∴ The car number is 1736.

Answer: 1736

9. In the unit's place addition, we have B + B + A giving A with a possible carry over. ∴ B must be 5 and the carry over from unit's place to tens place is 1.
Now, In the ten's place addition, we have carry over 1 + A +A + 5 giving A with a carry over C. This is possible only if A = 4 and C = 1.

The sum is 45 + 45 + 54 = 144. A = 4, B = 5 and C = 1 **Answer: 4**

10. 80 (Sum of the date numbers of Sundays = 2+ 9+ 16+23+30) See 2nd Table!

Sun	Mon	Tue	Wed	Thu	Fri	Sat
					1	2
3	4	5	6	7	8	9
10	**11**	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

Only one prime date on Monday.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
						1
2	**3**	4	5	6	7	8
9	10	11	12	13	14	15
16	**17**	18	19	20	21	22
23	24	25	26	27	28	29
30	**31**					

Here we have 3, 17 and 31 three prime dates on Monday. This works!

Note that in a 30-day month we will not get 6 diagonal entries as can be seen from the following table.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

Answer: 80

11. The following statements are made by the four children P; Q; R; S:
    - P says: Q, R, S are all girls.
    - Q says: P, R, S are all boys.
    - R says: Both P and Q are lying.
    - S says: P, Q, R are telling the truth.

    How many of the children were telling the truth?

    Solution: Only one

    S is definitely false as all of P,Q,R cannot be true

    If P is true, then Q is false; R is false and S is false.
    If Q is true, then P is false; R is false and S is false.
    If R is true, then P is false; Q is false and S is false.

    R could be true if P, Q are boys R, S are girls (one example)

    In any case, only one of them speak the truth. **Answer: 1**

12. **Solution:** We have 3 equations: $2A = 3D$; $2B = D + C + A$; $A = B + C$.
    As $C = A - B$ from the third equation, we get $2B = D + A - B + A$ on substituting in second equation. Thus $3B = D + 2A = D + 3D = 4D$. Substituting in equation 3, we get $C = A - B = 3D/2 - 4D/3 = D/6$. Hence 6 is the answer **Answer: 6**

13. Rajesh selects a 5-digit number. He gets a 4-digit number from this by erasing one of the digits. If the sum of the 5-digit and 4-digit numbers is 51185, then the sum of the digits of the 5-digit number is
    ---------.

    Solution: The 5-digit number is 46532. If we drop any other digit other than the units place the sum will be even. But the given sum is odd. Hence, we need to drop the units' place. This gives the above answer. The sum of the digits is 20.

    Answer: 20

14. The number of 3-digit numbers such that hundred's digit – ten's digit = ten's digit – unit's digit is ------

Solution: Let ABC be the 3-digit number. It is given that A – B = B – C which means A + C = 2B where A takes values 1, 2, 3, …, 9 and B and C take values from 0, 1, 2, …, 9. A and C have the same parity. If A is odd then C is also odd and the number of such ABC will be 5 x 5 = 25. If A is even then C is also even and the number of such ABC will be 4 x 5 = 20. Total will be 45 numbers. (Note that B is decided as (A + C)/2 once A, C are chosen.) **Answer: 45**

15. a, b, c are natural numbers such that a < b < c < 10 and $\frac{a}{b}+\frac{b}{c}=1$. The number of such combinations of triples a, b, c is ---------.

Solution: We need $\frac{a}{b}+\frac{b}{c}=1$, where a < b < c < 10. Hence, a cannot be 3 or more as the least values for b and c will be 4 and 5 leading to no solution.

a = 1, b = 2, and c = 4 is a solution; $\frac{1}{2}+\frac{2}{4}=1$

a = 2, b = 3, and c = 9 is a solution; $\frac{2}{3}+\frac{3}{9}=1$

a = 2, b = 4, and c = 8 is a solution; $\frac{2}{4}+\frac{4}{8}=1$

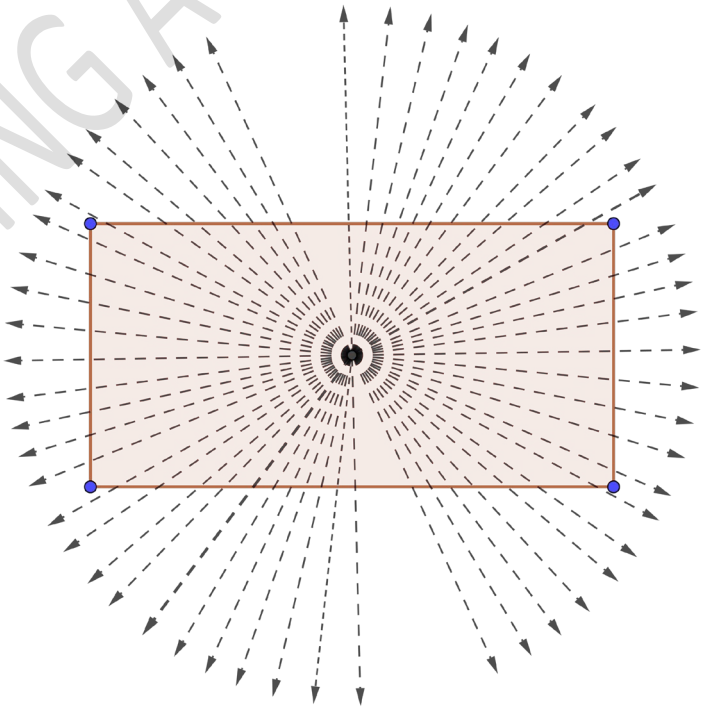
a = 2, b = 6, and c = 9 is a solution; $\frac{2}{6}+\frac{6}{9}=1$

Totally four solutions. **Answer: 4**

16. A rectangle needs to be cut into two parts of equal area by a straight line. How many such straight lines are possible? A. 4 B. 2 C. 6 D. infinitely many

Solution: The centre of the rectangle is the point of intersection of its diagonals. Any straight line through this point will divide the rectangle into two parts, in fact identical, with equal area. Hence there are infinitely many lines.

Here, you have few lines dividing the rectangle into equal areas!



Answer: D

17. Write down all the natural numbers which do not have the digit 3, in increasing order from 1: 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, ... ,21, 22, 24, …. What is the 300^{th} number in this sequence.

A. 520 B. 499 C. 474 D. 464

Solution: From 1 to 99, there are 19 numbers containing digit 3. Similarly, from 100 to 199 and 200 to 299, there are 19 numbers containing the digit 3. Thus from 1 to 299 we have 299 – 57 = 242 numbers without the digit 3. Numbers 300 to 399 will not come in the sequence. So, the 300^{th} number in the sequence is 58^{th} number from 400 in the sequence. It will occur in the block 470 to 479 as the fourth number as 54 numbers will be covered in 400 to 469. It will be 474.

Answer: C

18. Kaveri constructed a square on a grid of unit squares. She coloured all the 21unit squares placed along the diagonals of the constructed square. The total number of unit squares along the boundary of the constructed square, is ---------.

A) 36 B) 40 C) 44 D) 46

Solution: The number of squares colored by Kaveri is 21 (odd).
So, Kaveri must have colored 11 squares along each diagonal, the centre being common. 10+10+$\boxed{1}$=21. (centre square is boxed)

Therefore, the square on the grid should be made of 11×11 unit squares.

The total number of unit squares along the boundary of the constructed square, is $\boxed{1}$+9+$\boxed{1}$+9+$\boxed{1}$+9+$\boxed{1}$+9=40, (the corner squares are boxed)

Note: Each side has $\boxed{1}$+9+$\boxed{1}$=11 unit squares.

Answer: B

19. ABC, BCA, CAB are three different 3-digit numbers such that their sum is a 4-digit number with different digits. Then the 4-digit result is -------.

Solution: The sum of the digits in the three positions units, tens and hundreds is A + B + C. But the sum has four digits and all four digits are distinct. Hence the carry from ones to tens must be one and from tens to hundreds 2. For this to happen A+ B + C must be 19 giving the sum to be 2109.

Answer: 2109

20. Laurel and Hardy live in the same floor of a 40-storey building.
The ground floor and the basement are used for parking vehicles, garden, etc. . .
In each of the 40 floors, there were exactly 10 living apartments assigned with ten consecutive whole numbers as follows:
• In the 1st floor, the apartments are numbered 1 to 10.

• In the 2nd floor, the apartments are numbered 11 to 20.
• In the 3rd floor, the apartments are numbered 21 to 30.
and so on. . .
• In the 39th floor, the apartments are numbered 381 to 390.
• In the 40th floor, the apartments are numbered 391 to 400.

The sum of Laurel's floor number and Hardy's apartment number is 239.
Then, Hardy's apartment number is -------.

Solution: The floor number cannot be 21 as 21 + any apartment number in this floor (200 to 209) gives a number < 239.

Therefore, the floor number cannot be 21 or less.

The floor number cannot be 23 as 23 + any apartment number in this floor (220 to 229) gives a number > 239.

Therefore, the floor number cannot be 23 or more.

The only possible floor is the 22^{nd} floor.

The apartment number is 217 as 217 + 22 = 239.

Answer: 217